Frameless ALOHA with latency-reliability guarantees

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  - low peak throughput ($1/(2e)$ and $1/e$ respectively)
  - high packet loss rate even for low load

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- examples:
  - CRDSA [Cas-07]
  - IRSA [Liva-11]

[Cas-07] E. Casini, R. De Gaudenzi, and O. del Rio Herrero, "Contention Resolution Diversity Slotted ALOHA (CRDSA): An Enhanced Random Access Scheme for Satellite Access Packet Networks", IEEE Trans. on Wireless Commun., vol. 6, no. 4, pp. 1408-1419, April 2007

[Liva-11] G. Liva, "Graph-Based Analysis and Optimization of Contention Resolution Diversity Slotted ALOHA", IEEE Trans. Commun., vol. 59, no. 2, pp. 477-487, Feb. 2011

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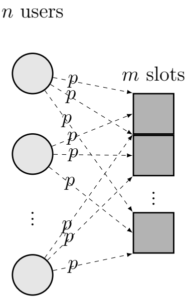
$$p = \frac{\beta}{n}$$



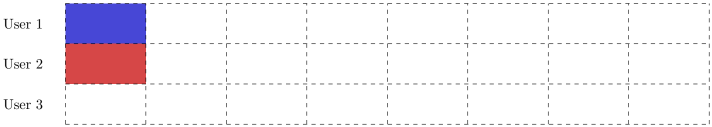
Frameless ALOHA
Example

contention starts

User 1								
User 2								
User 3								

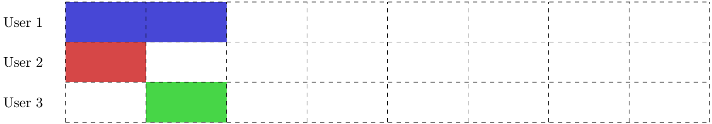
Frameless ALOHA
Example

slot 1



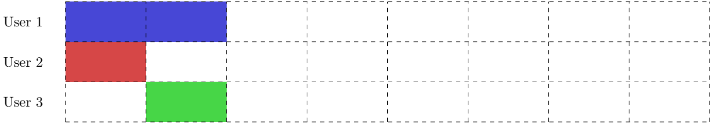
Frameless ALOHA
Example

slot 2



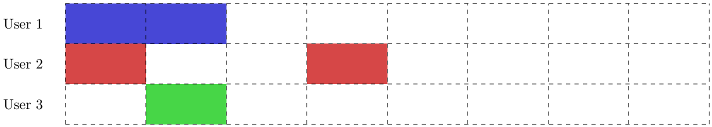
Frameless ALOHA
Example

slot 3



Frameless ALOHA
Example

slot 4



Frameless ALOHA
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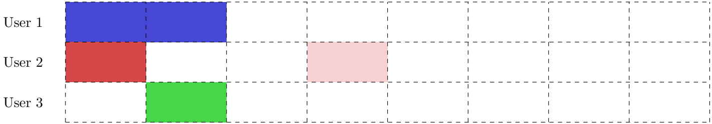
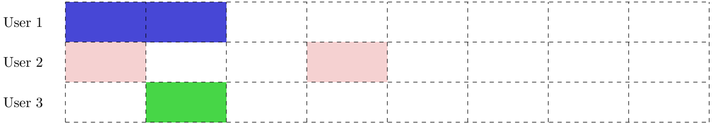
slot 4



decoding starts

Frameless ALOHA
Example

slot 4



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Frameless ALOHA
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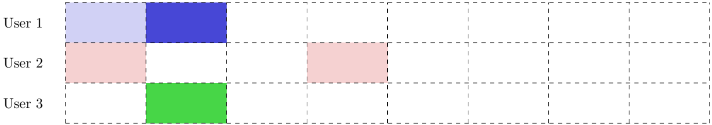

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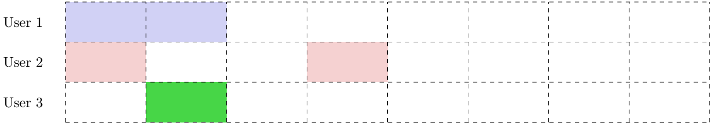

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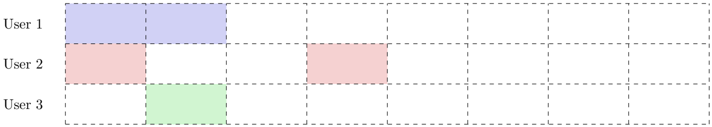


since all users are recovered we can terminate the contention after 5 slots

Finite Length Analysis
Definitions

Definition (Ripple)

We define the ripple as the set of singleton slots (reduced degree 1) and we denote it by $\mathscr{R}$.

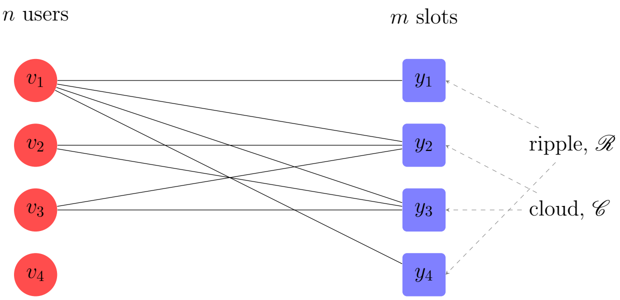
the cardinality of the ripple is denoted by r and its associated random variable as R.

Definition (Cloud)

We define the cloud as the set of slots with reduced degree $d \geq 2$ and we denote it by $\mathscr{C}$.

the cardinality of the cloud is denoted by c and the corresponding random variable as C.

Finite Length Analysis
Bipartite Graph Representation



Finite Length Analysis
Bipartite Graph Representation

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$$\Omega_i = \binom{n}{i} p^i (1-p)^{n-i}.$$

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  - at every iteration:
    - if there are singleton slots $\rightarrow$ one user is resolved
    - if there are no singleton slots decoding stops

Finite Length Analysis
Finite State Machine

- the iterative SIC process can be modelled by means of a **finite state machine** with state:

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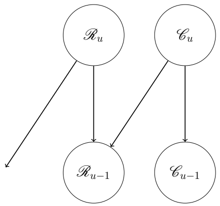
$$\mathtt{S}_u := (\mathtt{C}_u, \mathtt{R}_u)$$

  - $\mathtt{C}_u$: cardinality of cloud when u users are unresolved
  - $\mathtt{R}_u$: cardinality of ripple when u users are unresolved

Finite Length Analysis
Transition from u to $u-1$ resolved users



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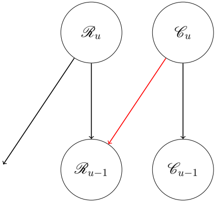


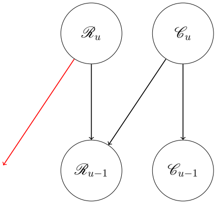
- $\mathtt{b}_u$: the number of slots leaving $\mathscr{C}_u$ and entering $\mathscr{R}_{u-1}$

$$\mathtt{b}_u := \mathtt{c}_u - \mathtt{c}_{u-1}$$

- B_u: associated random variable

Finite Length Analysis

Transition from u to $u-1$ resolved users



- $\mathtt{a}_u$: the number of slots leaving the ripple $\mathscr{R}_u$ in the transition
- $\mathtt{A}_u$: r.v. associated to $\mathtt{a}_u$

Finite Length Analysis
Main Result

Theorem

Given that the decoder is at state $\mathtt{S}_u = (\mathtt{c}_u, \mathtt{r}_u)$*, when* u *users are unresolved and with* $\mathtt{r}_u > 0$*, the probability of the decoder being at state* $\Pr\{\mathtt{S}_{u-1} = (\mathtt{c}_{u-1}, \mathtt{r}_{u-1})\}$ *when* $u-1$ *users are unresolved is given by*

$$\Pr\{\mathtt{S}_{u-1} = (\mathtt{c}_u - \mathtt{b}_u, \mathtt{r}_u - \mathtt{a}_u + \mathtt{b}_u) | \mathtt{S}_u = (\mathtt{c}_u, \mathtt{r}_u)\} = \binom{\mathtt{c}_u}{\mathtt{b}_u} {q_u}^{\mathtt{b}_u} (1-q_u)^{\mathtt{c}_u - \mathtt{b}_u} \binom{\mathtt{r}_u - 1}{\mathtt{a}_u - 1} \times \left(\frac{1}{u}\right)^{\mathtt{a}_u - 1} \left(1 - \frac{1}{u}\right)^{\mathtt{r}_u - \mathtt{a}_u}$$

for $0 \le \mathtt{b}_u \le \mathtt{c}_u$*,* $\mathtt{b}_u - \mathtt{a}_u \le \mathtt{r}_u$ *and* $\mathtt{a}_u \ge 1$*, and with*

$$q_u = \frac{\sum_{d=2}^{n-u-2} \Omega_d \, d(d-1) \frac{1}{n} \frac{u-1}{n-1} \frac{\binom{n-u}{d-2}}{\binom{n-2}{d-2}}}{1 - \sum_{d=1}^{n-u-1} \Omega_d \, u \frac{\binom{n-u}{d-1}}{\binom{n}{d}} - \sum_{d=0}^{n-u} \Omega_d \frac{\binom{n-u}{d}}{\binom{n}{d}}}.$$

Finite Length Analysis
Throughput and PER

- In practice one is interested in the packet error rate P_e

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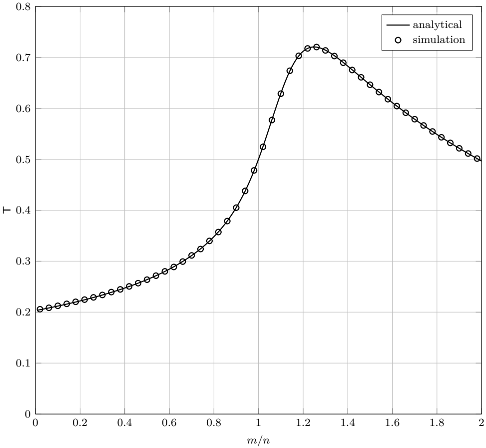
- The throughput T is the number of resolved users normalized by the number of slots:

$$\mathsf{T} = \frac{n(1-\mathsf{P_e})}{m} = \frac{1-\mathsf{P_e}}{m/n}.$$

Finite Length Analysis
Throughput for $\beta = 2.5$, for $n = 100$



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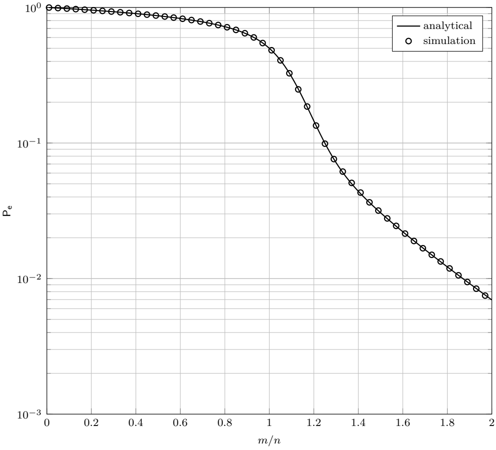

Latency-Reliability Guarantees
Motivation

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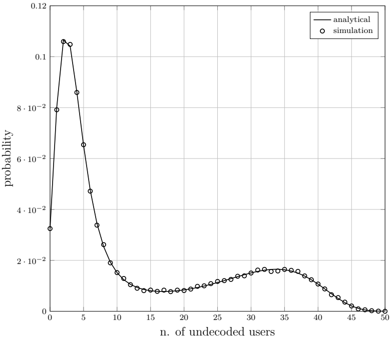
  - Example: $n = 50$ users, $m = 100$ slots



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- this value of P_k might be too high for many applications

Latency-Reliability Guarantees
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- the analysis needs to be modified
  - the iterative SIC process is modelled by means of a finite state machine with state:
    $$\mathtt{S}_u := (\mathtt{C}_u^{(1)}, \mathtt{C}_u^{(2)}, \ldots, \mathtt{C}_u^{(h)}, \mathtt{R}_u)$$
  - $\mathtt{C}_u^{(i)}$: cardinality of i-th cloud when u users are unresolved (number of slots of type i with reduced degree 2 or larger)
  - $\mathtt{R}_u$: cardinality of ripple when u users are unresolved

Latency-Reliability Guarantees

Optimization - Results

- how much can we improve the performance?

Latency-Reliability Guarantees
Optimization - Results

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slot classes	P_k	parameters
1	0.9334	$\beta = 3.33$, $m = 100$
2	0.9986	$\beta_1 = 2.53$, $m_1 = 86$ $\beta_2 = 22.08$, $m_2 = 14$
3	0.99917	$\beta_1 = 2.51$, $m_1 = 88$ $\beta_2 = 17.39$, $m_2 = 11$ $\beta_3 = 50$, $m_3 = 1$
dynamical	0.9999975	dynamical

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- the analysis yields the exact probability of meeting a **latency-reliability** target
- application example: introducing different slot classes we can decrease the probability of not meeting a latency-reliability target by almost 2 orders of magnitude
- a dynamical strategy can provide even further gains (2 orders of magnitude better than static).