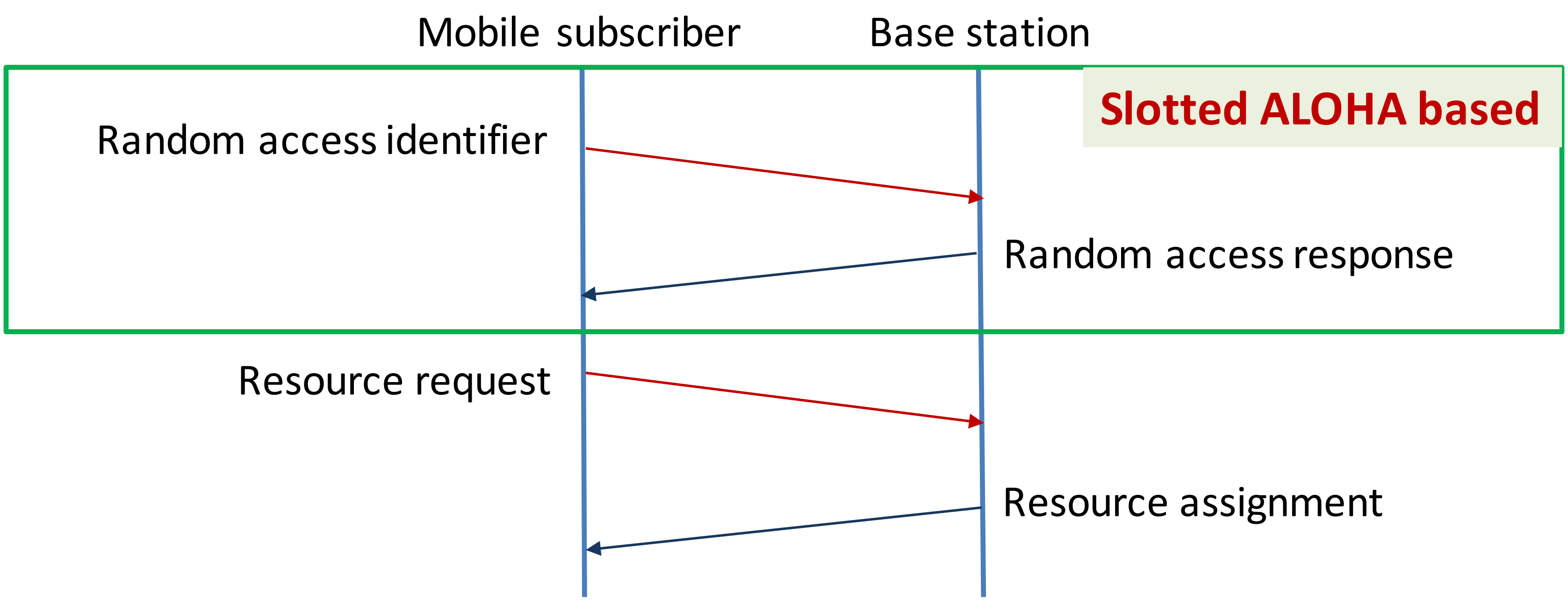
# Slotted ALOHA

Čedomir Stefanović

# Cellular access protocols

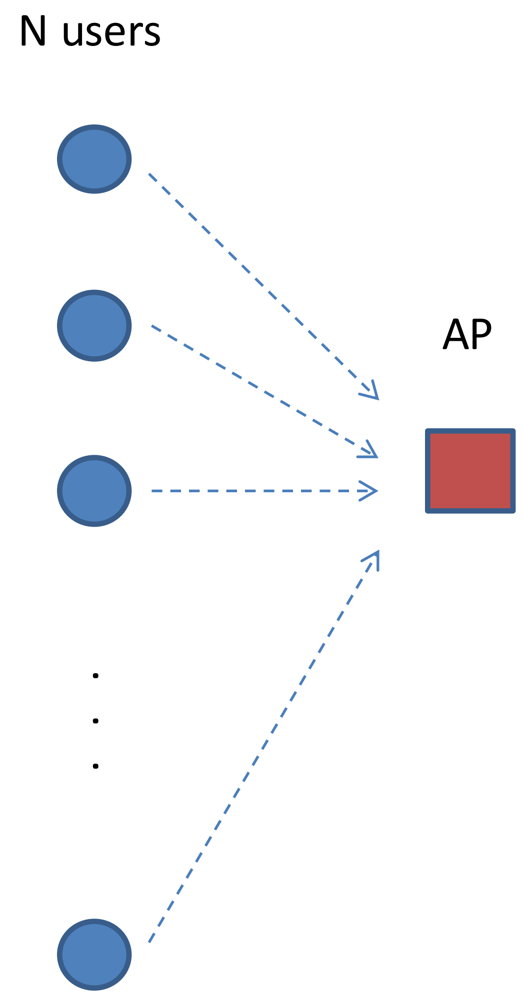
- All (mobile) cellular standards use a similar algorithm for the initial connection establishment
- Access Reservation Protocol:



# Slotted ALOHA

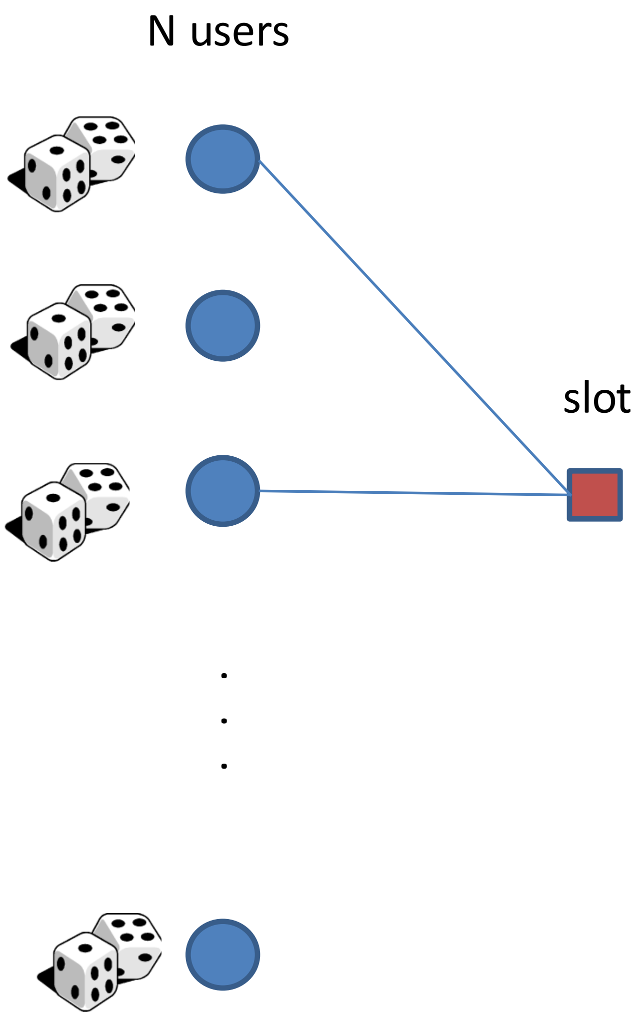
- L. G. Roberts, "Aloha packet system with and without slots and capture," SIGCOMM Comput. Commun. Rev., Apr. 1975.

- $N$ users, single access point (AP)
  - Homogenous population
  - Equal length packets
- Random access
  - Distributed, decentralized
  - User behave in the same way
- Link time is divided in slots of equal duration
  - Slot-synchronization of the users is assumed
- Users contend for the access to the base station



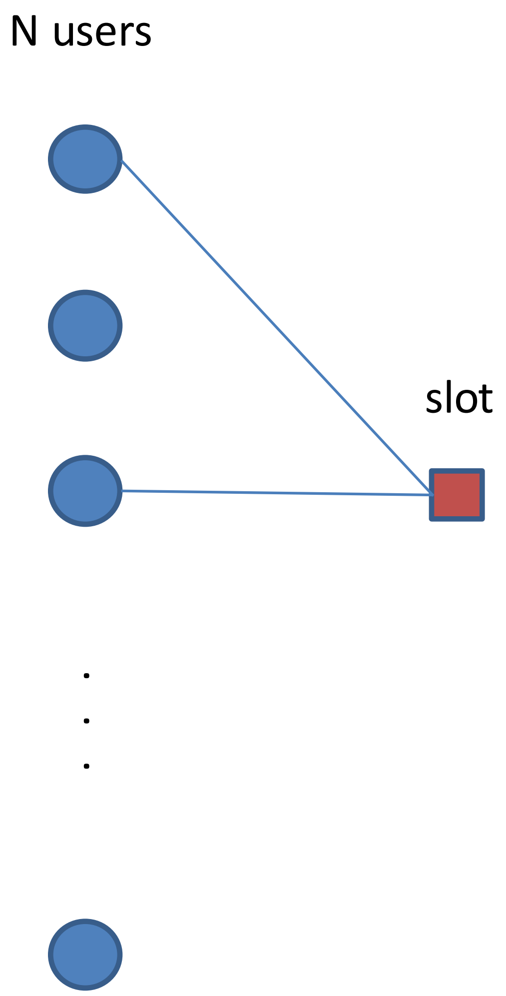
# Slotted ALOHA

- Each users contends (transmits) with a predefined probability $p_A$
- A slot can be:
  - Idle
  - Singleton (i.e., containing single transmission)
  - Collision (containing multiple transmissions)
    - Colisions are destructive!
- Feedback after every slot
  - Unsuccessful users contend in the next slot with (possibly changed) $p_A$



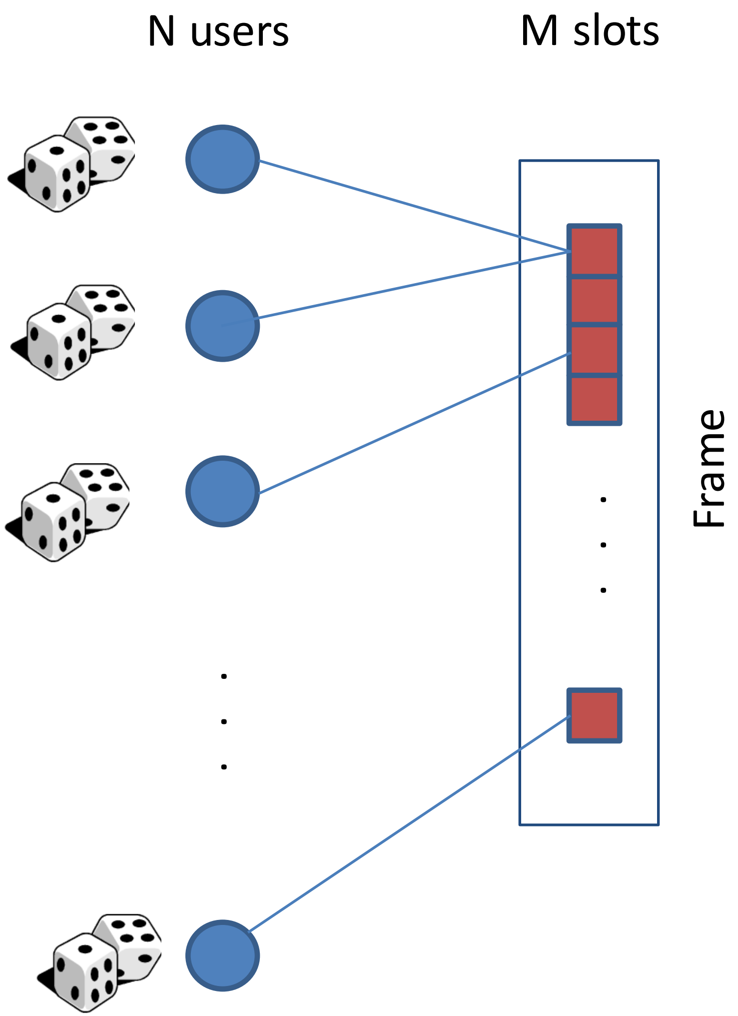
# Slotted ALOHA

- Throughput:
  - Measure of efficiency of use of system resources (slots)
  - Average fraction of slots with successful transmission attempts
    - I.e., fraction of singleton slots
- Probability that a slot is a singleton:
  - $T = \binom{N}{1} p_A (1-p)^{N-1} \approx N p_A e^{-N p_A}$
  - $T_{max} = \frac{1}{e} \approx 0.37$ (when $N p_{\mathrm{A}} = 1$)



# Framed slotted ALOHA

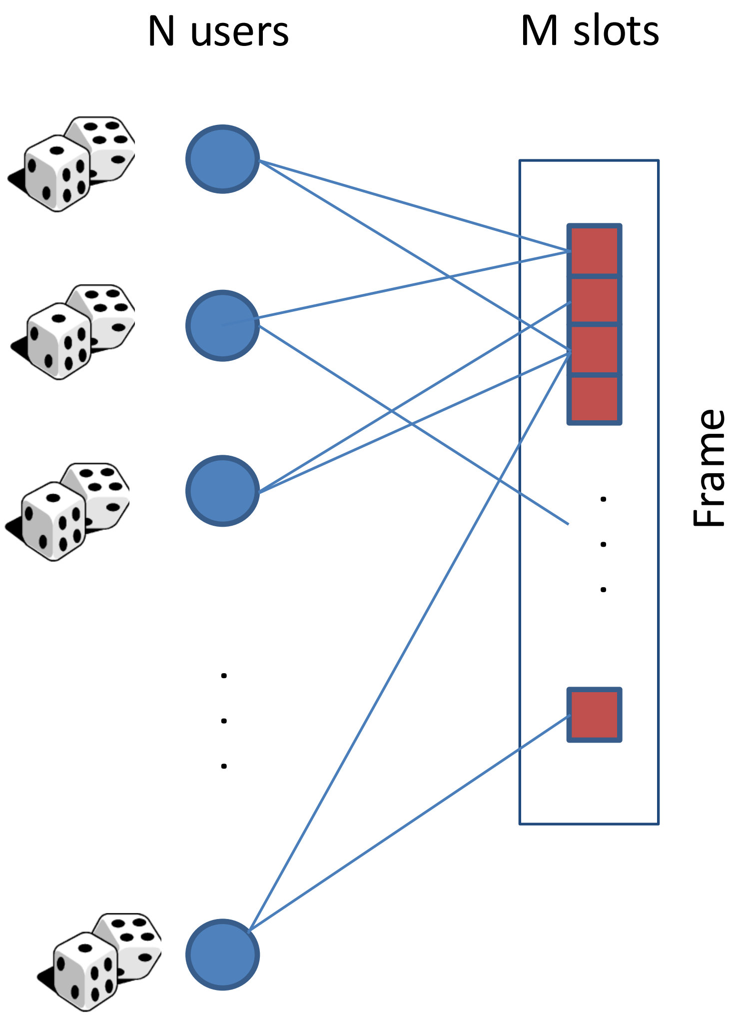
- H. Okada, Y. Igarashi, Y. Nakanishi, "Analysis and application of framed ALOHA channel in satellite packet switching networks", Electronics and Communications, 1977
- Each users transmits just once in a randomly selected slot of the frame
- Again, only singleton slots are useful
- Feedback on a frame basis
- Throughput:
  - Probability that a slot is a singleton:
- $T = \binom{N}{1}\left(\frac{1}{M}\right)\left(1-\frac{1}{M}\right)^{N-1} = \frac{N}{M}e^{-\frac{N}{M}}$
  - $T_{max} = \frac{1}{e} \approx 0.37$ (when $\frac{N}{M} = 1$)



# SLOTTED ALOHA BEYOND THE COLLISION MODEL

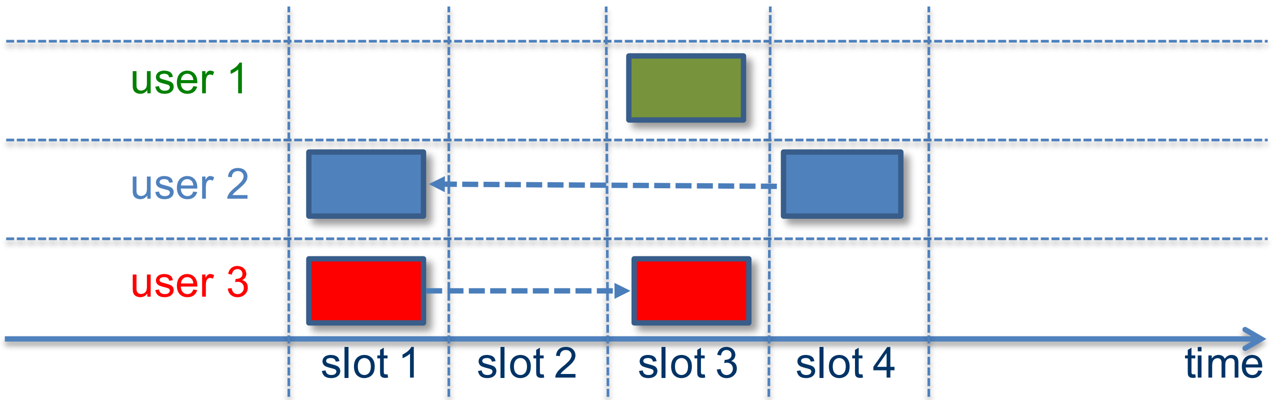
# Contention resolution diversity slotted ALOHA

- E. Casini, R. De Gaudenzi, O. del Rio Herrero, "Contention Resolution Diversity Slotted ALOHA (CRDSA): An Enhanced Random Access Scheme for Satellite Access Packet Networks", IEEE Trans. Wireless Communications, 2007
- Users repeat their transmission in several randomly chosen slots of the frame
  - Same number of replicas per user
- Collisions can be exploited!
  - Successive interference cancellation (SIC)
  - Improves throughput
  - $T \approx 0.55$ for CRDSA with two repetitions per user



# SIC in slotted ALOHA-based protocols

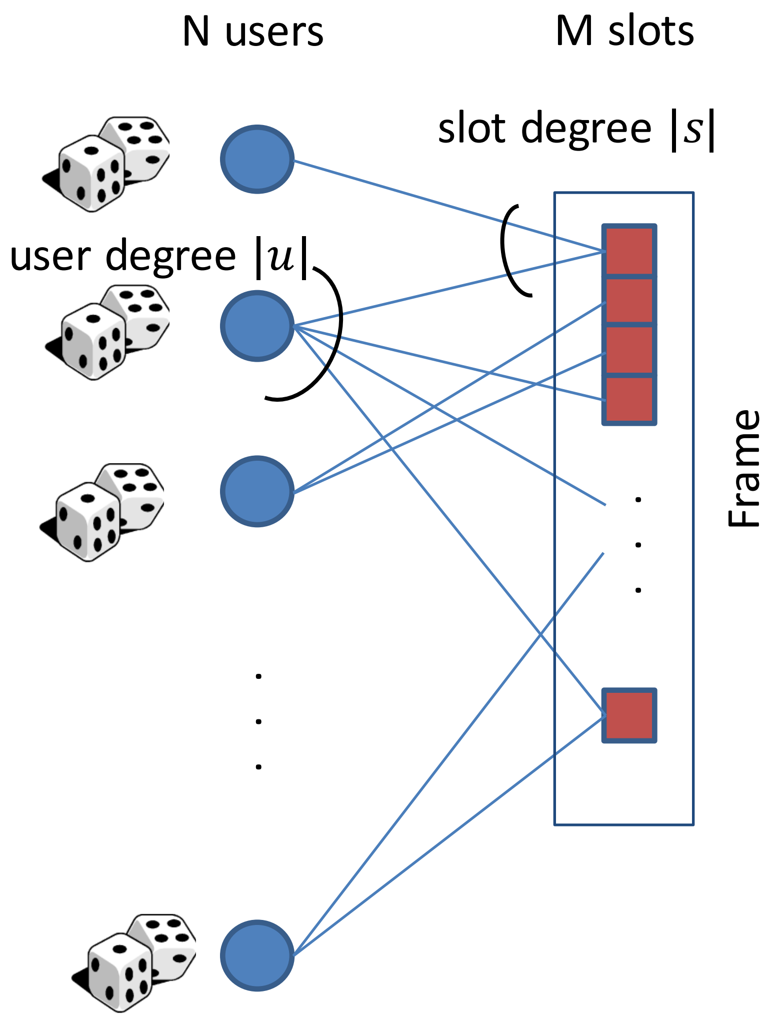
- Each successfully decoded replica enables canceling (removal) of other replicas




- In the first approximation, it is assumed that interference cancellation is perfect
  - Valid in certain systems – satellite communications with moderate to high SNR

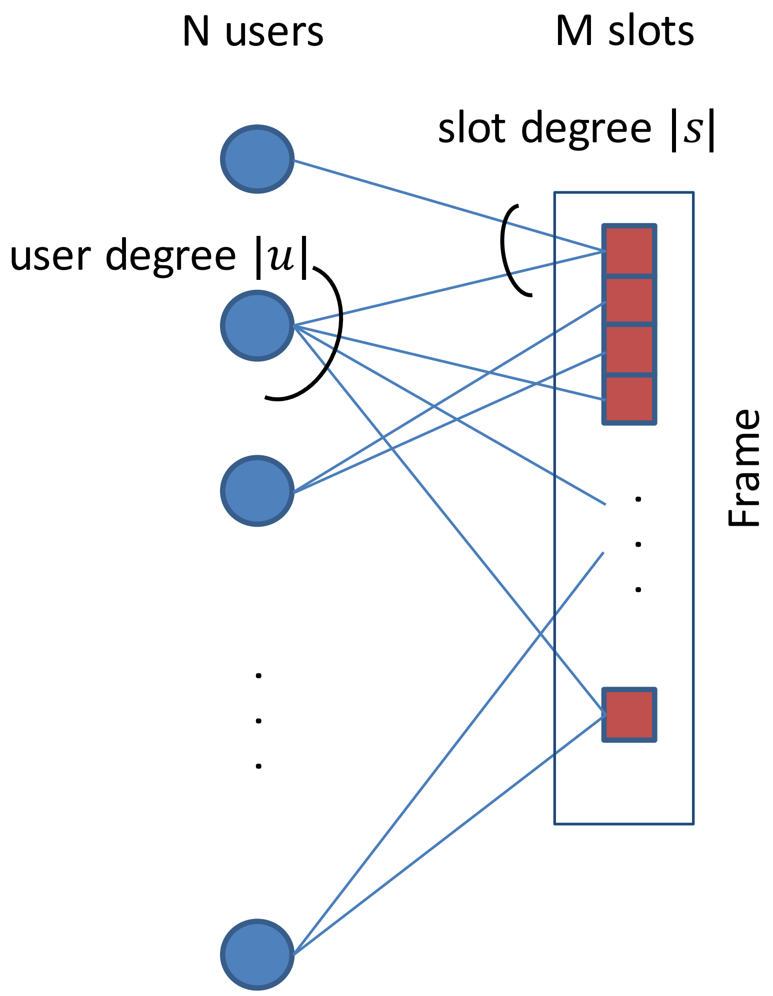
# Irregular repetition slotted ALOHA

- G. Liva, "Graph-Based Analysis and Optimization of Contention Resolution Diversity Slotted ALOHA," IEEE Trans. Communications, 2011.
- Generalization of CRDSA
  - Repetition rate can vary across users
  - Every user selects its no. of repeated transmissions according to a predefined distribution
- Interference cancellation (IC) can be seen as iterative procedure performed on the graph
  - Resembles decoding procedure of erasure correcting codes
- IC in frame-slotted ALOHA – analogous to fixed-rate block error-correction coding

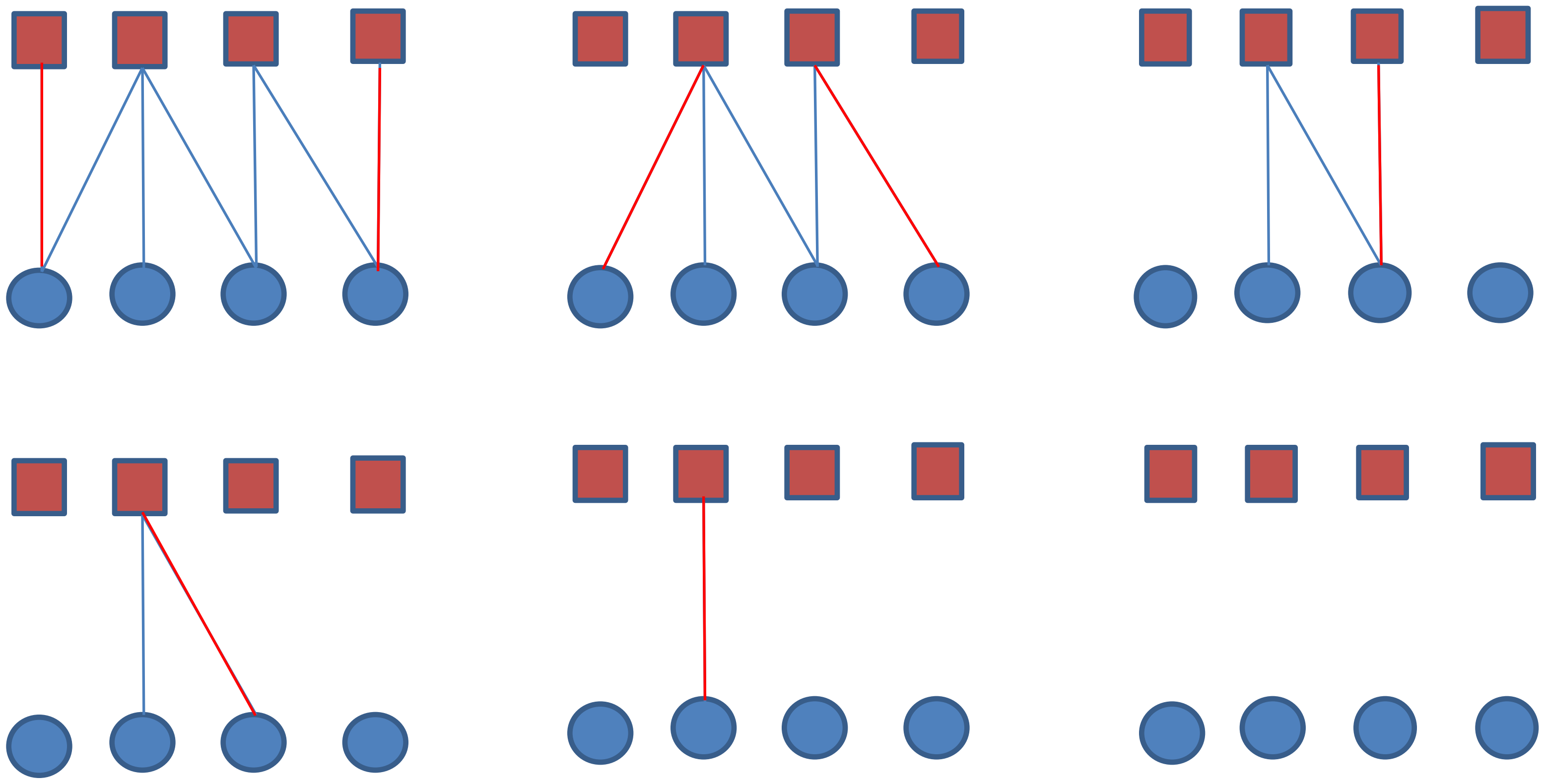


# Irregular repetition slotted ALOHA

- Left degrees can be directly controled
- Right degrees can be only indirectly controlled
  - Poisson distributed with “indirectly controllable” mean
- Optimal repetition strategies (i.e., optimal user degrees) are drawn according to the distributions that are used for left-irregular LDPC codes
- Plenty of subsequent results:
  - Analogies with LDPC codes, optimal distributions
  - Capacity bounds
  - Asymptotically throughput tends to 1, but modest throughputs for more realistic number of users (50 - 1000)

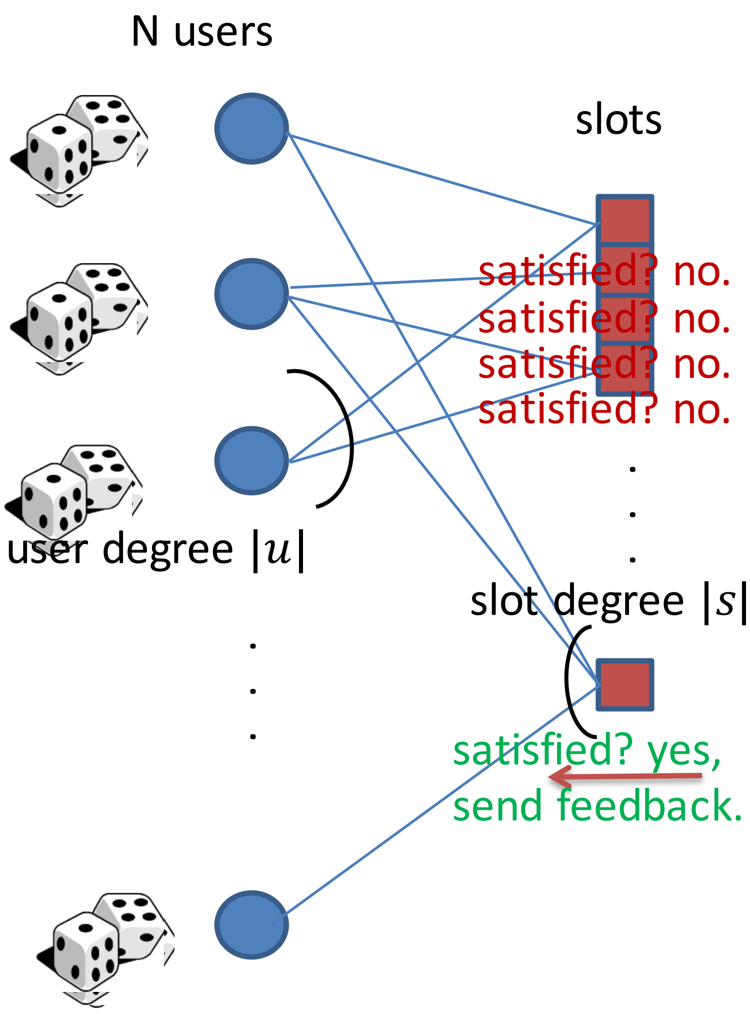


# Successive interference cancellation



# Frameless ALOHA

- C. Stefanovic, P. Popovski, D. Vukobratovic, “Frameless ALOHA Protocol for Wireless Networks”, IEEE Communication Letters, Dec. 2012
- Idea: Apply paradigm of rateless codes to slotted ALOHA:
  - No predefined frame length
  - Slots are successively added until a criterion related to performance parameters of the scheme is satisfied
  - Optimization of the slot-access probability and termination criterion



# Frameless ALOHA: Optimization of the slot access probability

- The simplest case:
  - All users use the same slot access probability $p_a$ for all the slots

$$p_a = \frac{\beta}{N}$$

  - $\beta$ is the average slot degree

- Goal: Maximize throughput $T$

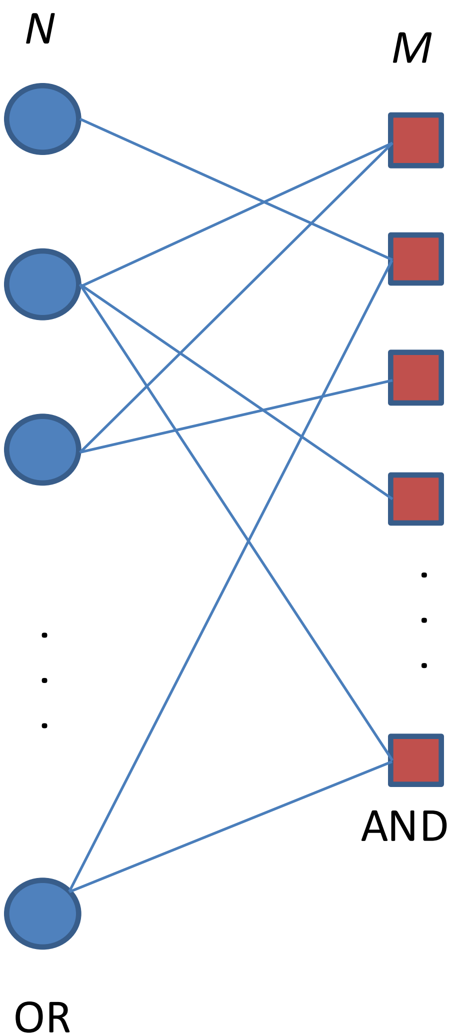
$$T = \frac{N_R}{M} = \frac{P_R\, N}{M}$$

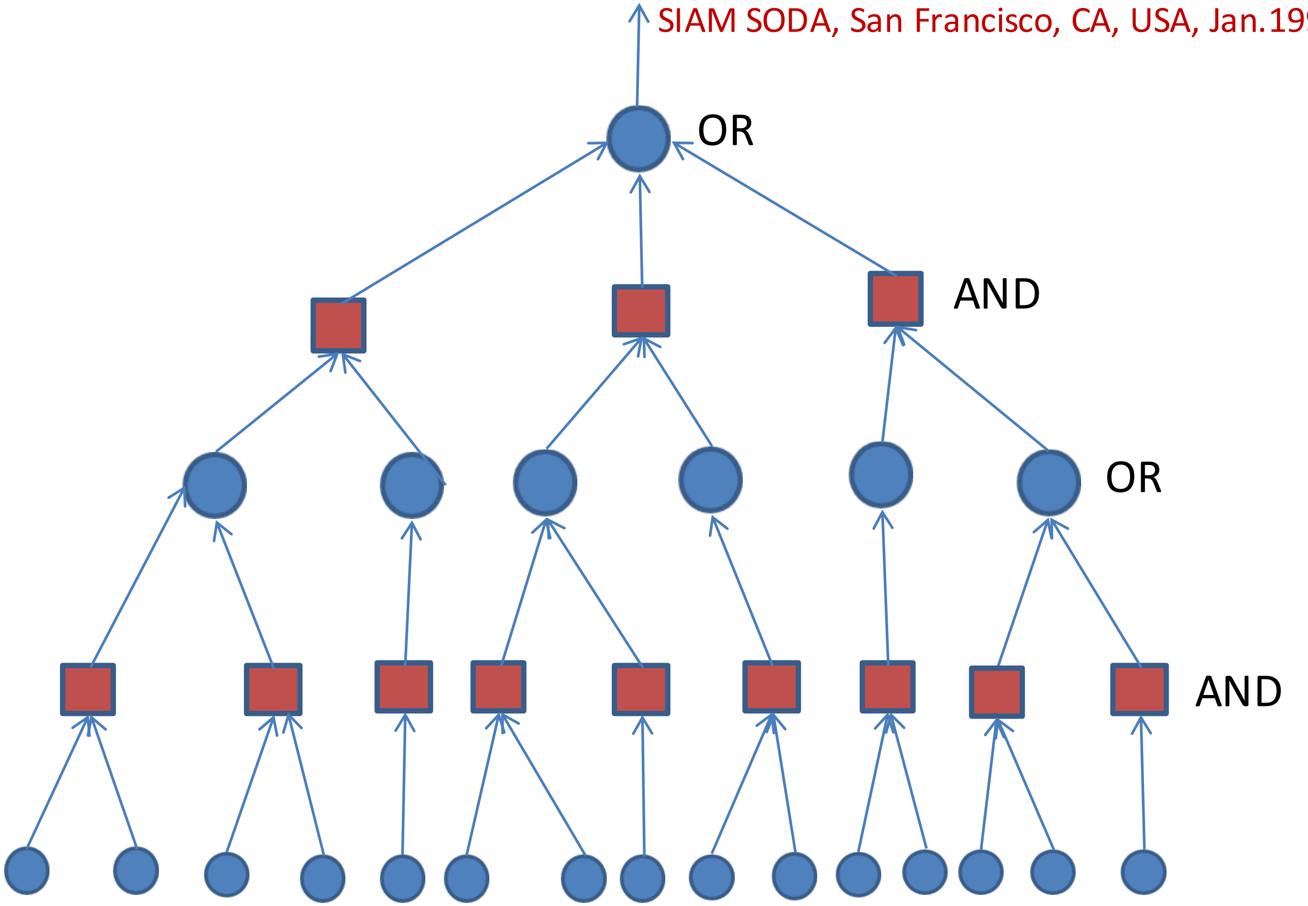
  - $N_{\mathrm{R}}$ is the number of resolved users (transmissions)
  - $P_R$ is the probability of user resolution

- Select $\beta$ such that throughput is maximized

# And-or tree evaluation: Asymptotic analysis tool

M. G. Luby , M. Mitzenmacher, M. A. Shokrollahi, "Analysis of Random Processes via And-Or Tree Evaluation", in Proc. of 9th ACM-SIAM SODA, San Francisco, CA, USA, Jan.1998

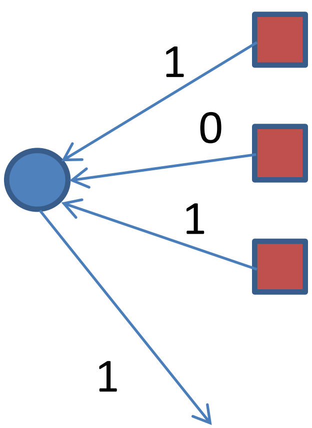


# And-or tree evaluation: Message update rules

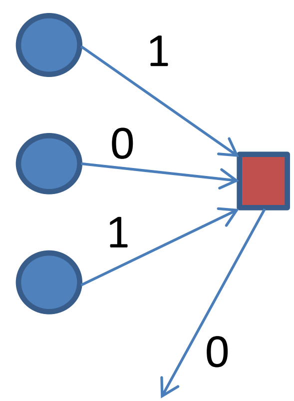
## OR nodes

- Outgoing message is 1 if any of the incoming messages is 1
  - Message 1 = user (transmission) decoded




## AND nodes

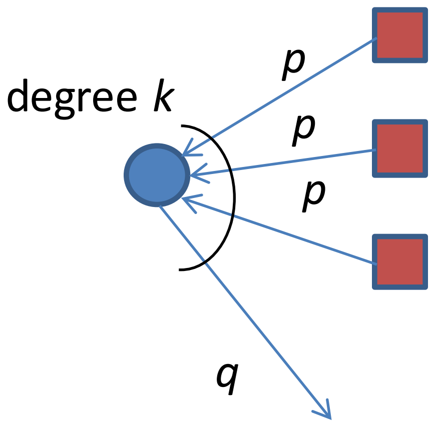
- Outgoing message is 1 if all incoming messages are 1
  - Message 1 = user decoded



# And-or tree evaluation: Message update probabilities

**OR nodes**

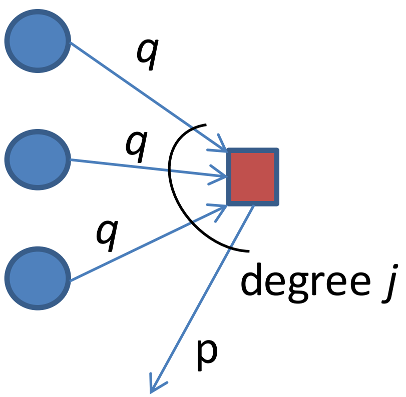
- $p$ – probability that the value of the incoming message is zero
- $q$ – probability that the value of the outgoing message is zero




$$q = p^{k-1}$$

**AND nodes**

- $p$ – probability that the value of the outgoing message is zero
- $q$ – probability that the value of the incoming message is zero




$$p = 1 - (1 - q)^{j-1}$$

# And-or tree evaluation: Message update probabilities

**OR nodes**

- The expected (i.e., average) probability that the outgoing message is 0 is:
- $q = \sum_k \lambda_k p^{k-1} = \lambda(p)$
- where:
  - $\lambda_k$ - probability that message is egressing a node of degree $k$, $\sum_k \lambda_k = 1$
  - $\lambda(x) = \sum_k \lambda_k x^{k-1}$

**AND nodes**

- The expected (i.e., average) probability that the outgoing message is 0 is:
- $p = \sum_j \omega_j (1 - (1-q)^{j-1})$
- $= 1 - \omega(1-q)$
- where:
  - $\omega_j$ - probability that message is egressing a node of degree $j$, $\sum_j \omega_j = 1$
  - $\omega(x) = \sum_j \omega_j x^{j-1}$

edge-oriented degree distributions

# And-or tree evaluation

$$q(i) = \lambda\Big(1 - \omega\big(1 - q(i-1)\big)\Big)$$

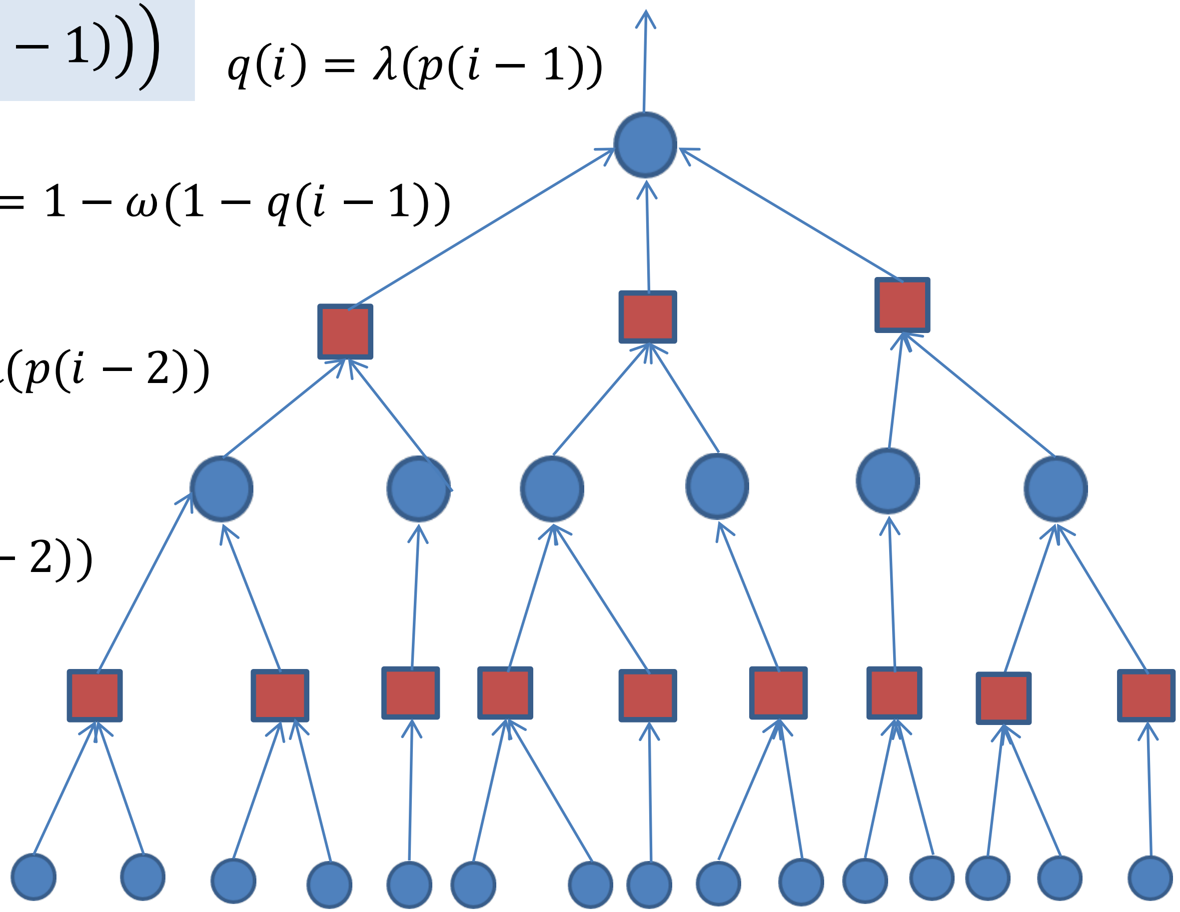
$$q(i) = \lambda(p(i-1))$$

$$p(i-1) = 1 - \omega(1 - q(i-1))$$

$$q(i-1) = \lambda(p(i-2))$$

$$p(i-2) = 1 - \omega(1 - q(i-2))$$

$$q(i-2)$$



# And-or tree evaluation: Performance parameters

- And-or tree evaluation shows the expected asymptotic performance based on the statistical graph description expressed through $\lambda(x)$ and $\omega(x)$

- Probability of user resolution:

$$P_R = 1 - \lim_{i \to \infty} q(i)$$

  - with the initial value $q(0) = 1$

- Asymptotic throughput is:

$$T = \frac{P_R N}{M}$$

# And-or tree evaluation: Deriving the slot degree distribution

- Slot $s$
  - Slot access probability $p_{\mathrm{A}}$ (same for all users)
  - $p_{\mathrm{A}} = \frac{\beta}{N}$
  - $\beta$ is the average degree of slot $s$ (load of slot $s$)
    - Actual degree of slot $s$ is determined by binomial distribution (which can be approximated by Poisson distribution):
    - $P[|s| = k\,] = \binom{N}{k}(p_{\mathrm{A}})^k\,(1-p_{\mathrm{A}})^{N-k} \approx \frac{\beta^k}{k!}e^{-\beta} = \Omega_k$
- Slot degree distribution:
  - $\Omega(x) = \sum_k \Omega_k x^k = e^{-\beta(1-x)}$
- Edge-oriented degree distribution:
  - $\omega_k = \frac{k\,\Omega_k}{\sum_j j\,\Omega_j}$
  - $\omega(x) = \sum_k \omega_k x^k = e^{-\beta(1-x)}$

# And-or tree evaluation: Deriving the user degree distribution

- User $u$
  - Assume that there are $M$ slots

$$P[|\mathrm{u}| = k\,] = \binom{\mathrm{M}}{k} (p_{\mathrm{A}})^k\,(1 - p_{\mathrm{A}})^{M-k} \approx \frac{(Mp_A)^k}{k!} e^{-Mp_A} =$$

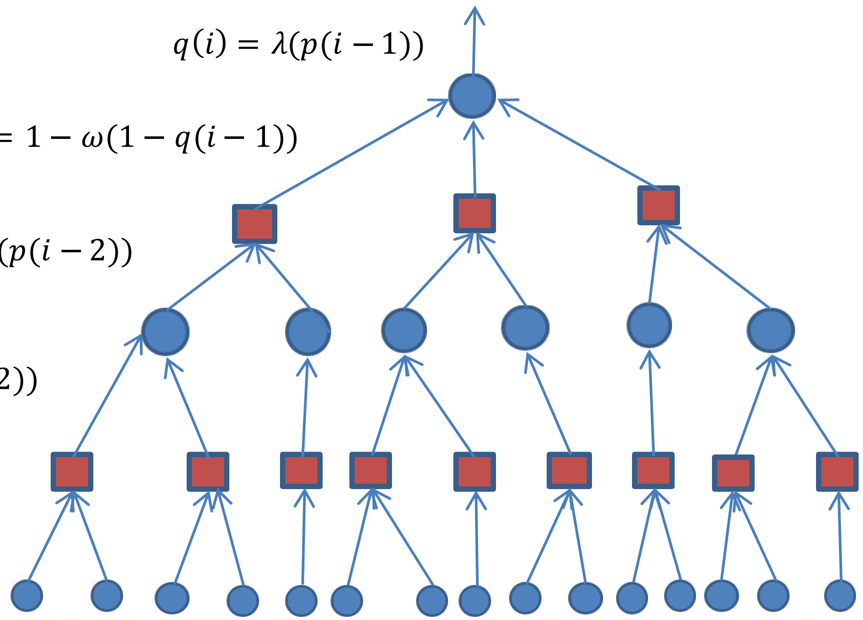
$$= \frac{\big((1+\epsilon)\beta\big)^k}{k!} e^{-(1+\epsilon)\beta} = \Lambda_k$$

    - where $(1+\epsilon) = \frac{M}{N}$

- User degree distribution:
  - $\Lambda(x) = \sum_k \Lambda_k x^k = e^{-(1+\epsilon)\beta(1-x)}$
- Edge-oriented degree distribution:
  - $\lambda_k = \frac{k\,\Lambda_k}{\sum_j j\,\Lambda_j}$
  - $\lambda(x) = \sum_k \lambda_k x^k = e^{-(1+\epsilon)\beta(1-x)}$

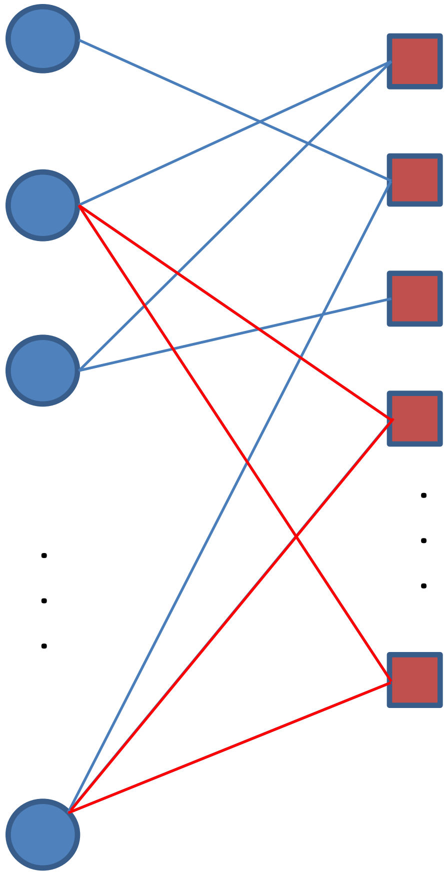
# And-or tree evaluation: Message update probabilities

- $q_i = \lambda(p_{i-1}) = e^{-(1+\epsilon)\beta(1-p_{i-1})}$
- $p_i = 1 - \omega(1 - q_i) = 1 - e^{-\beta(1-q_i)}$



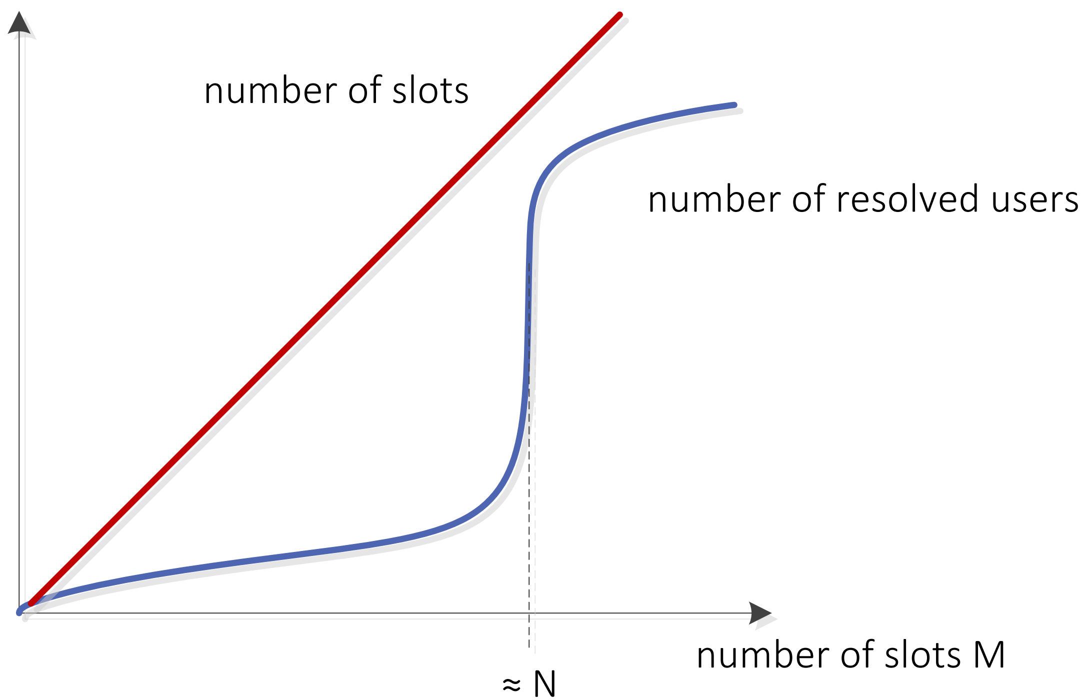
# And-or tree evaluation

- Our graphs are not trees!
  - There are loops
    - i.e., there are interdependencies among messages
  - The results obtained by the and-or tree evaluation pose upper limits on the performance

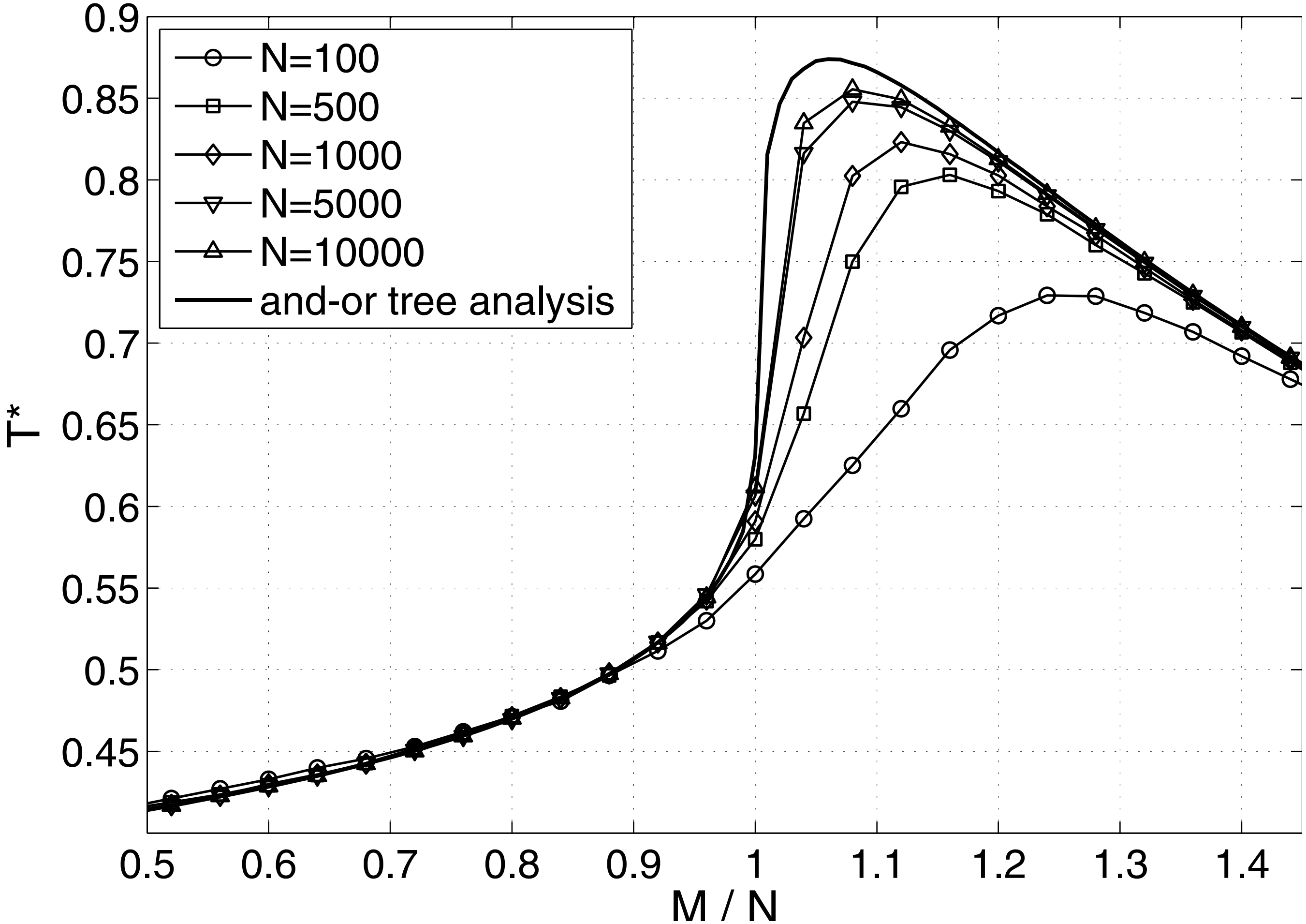


# Optimizing the slot access probability: Results

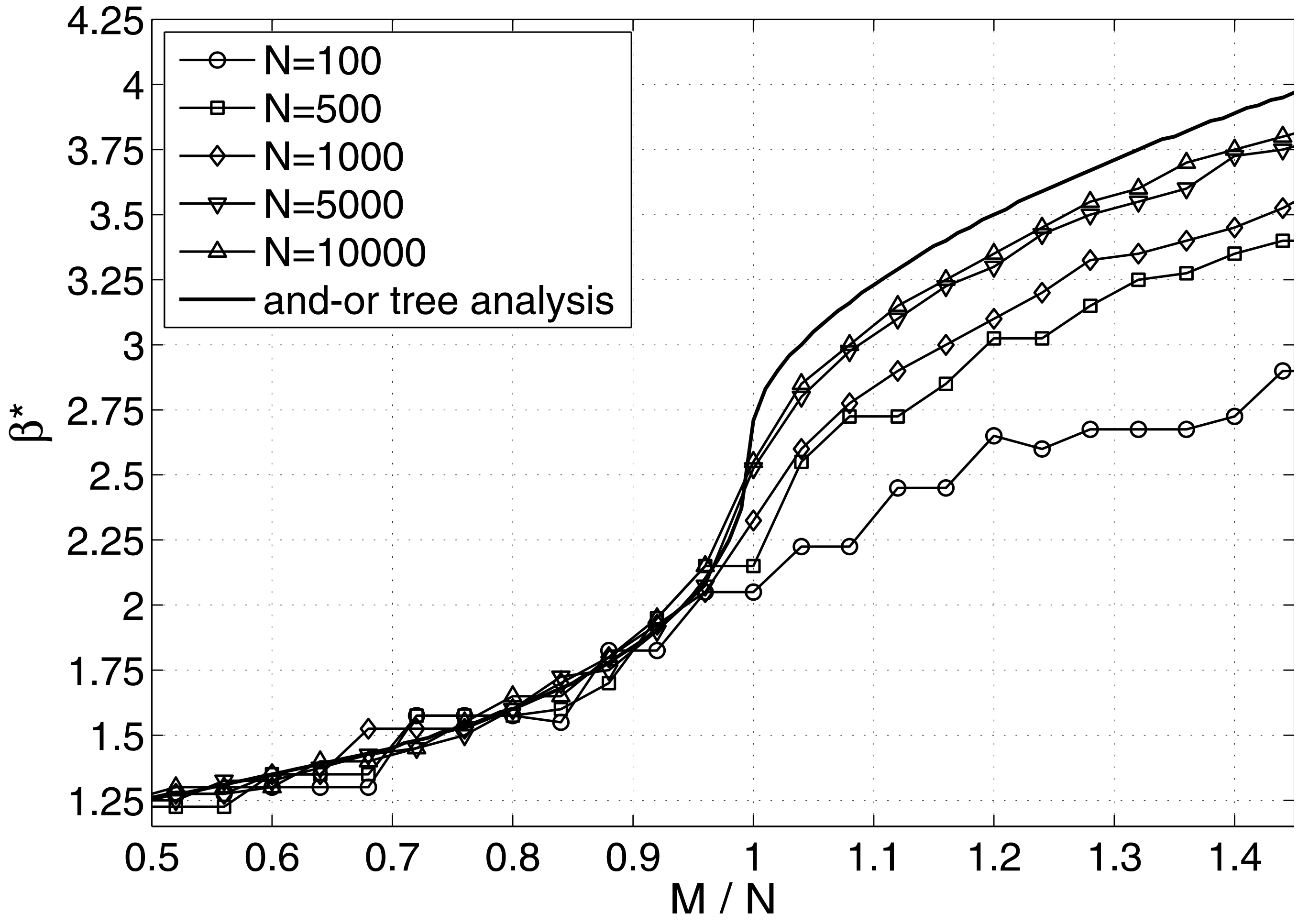
- All slots have the same expected degree $\beta$ :
  - Asymptotic analysis ($N \to \infty$) – and-or tree evaluation
  - Simulations for $N = 100{,}500{,}1000, 5000, 10000$



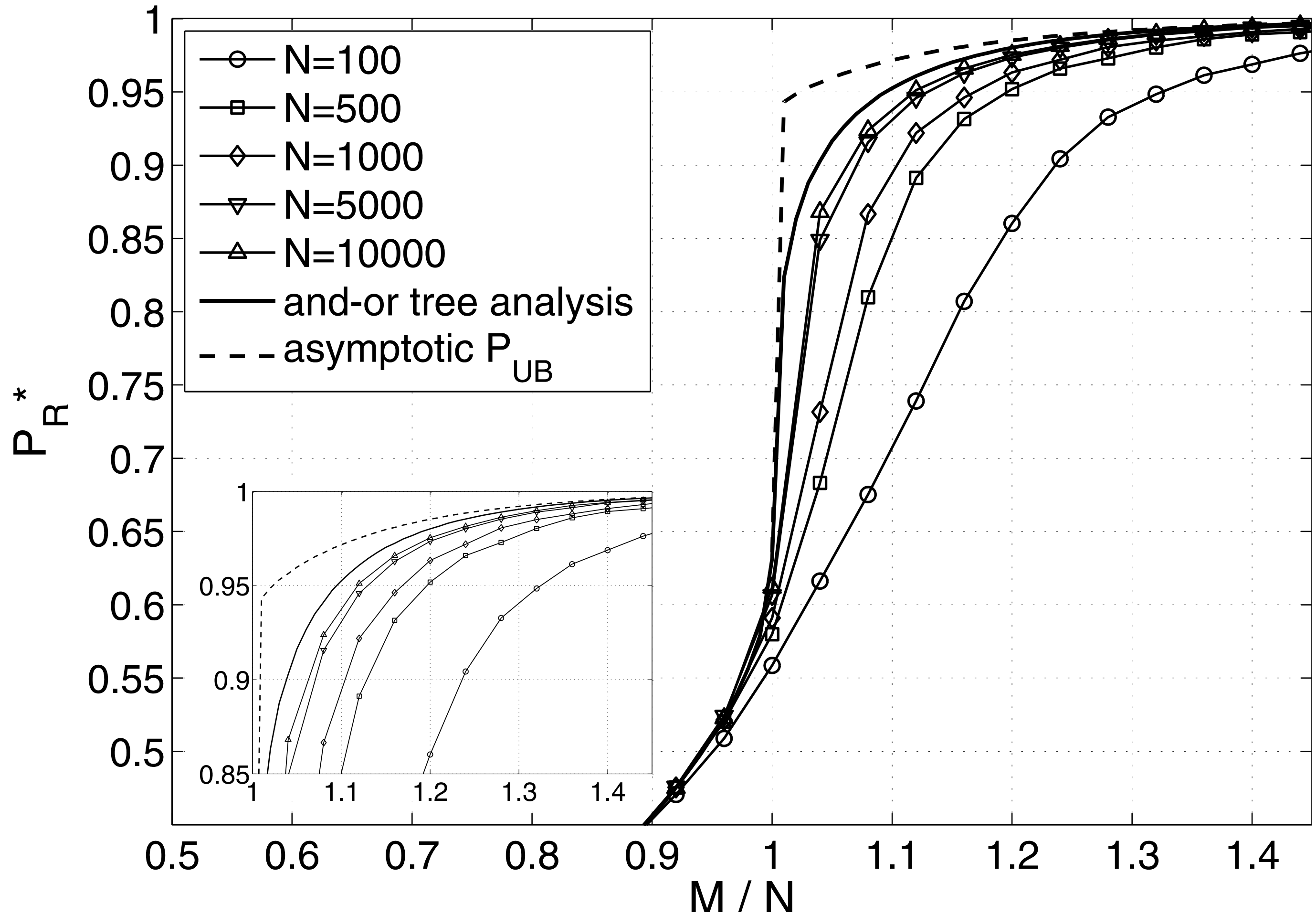
# Maximal throughput



# Optimal slot degree (which yields maximal throughput)

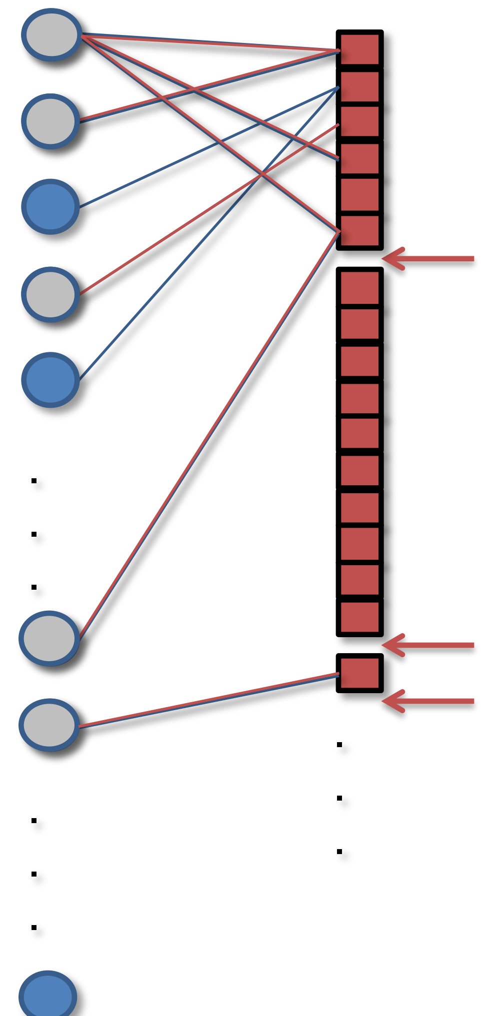


# Probability of user resolution



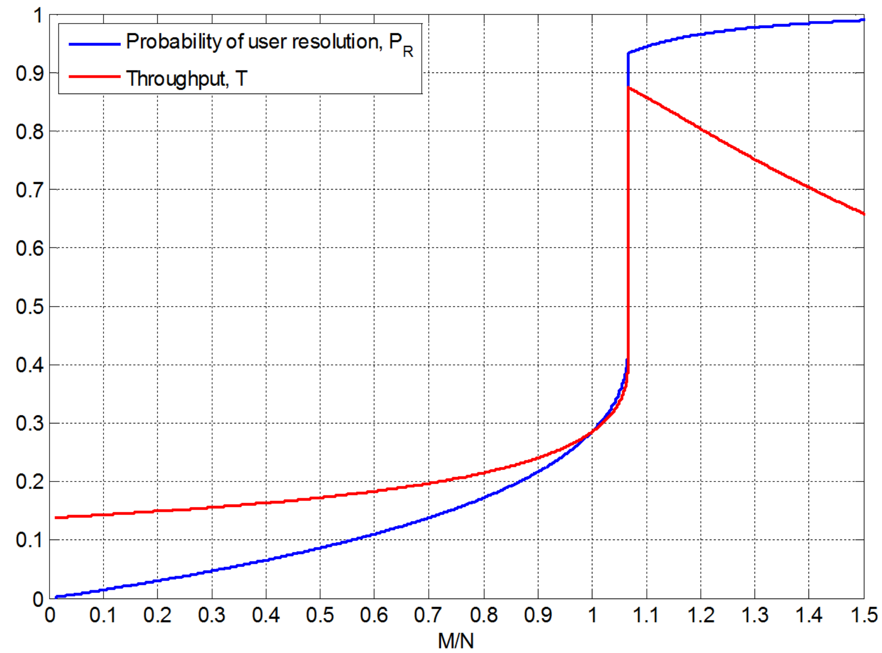
# Frameless ALOHA: Optimizing the stopping criterion

- C. Stefanovic, P. Popovski, "ALOHA Random Access that Operates as a Rateless Code", IEEE Trans. Communications, Nov. 2013
- Single feedback used after $M$-th slot
  - $M$ not defined in advance
  - Analogous to rateless coding framework!
- After feedback, new contention period
- When to send feedback?
  - E.g., when the throughput is high enough (ideally the highest possible)



# Frameless ALOHA: Optimizing the stopping criterion

- The graph shows the evolution of the probability of user resolution $P_R$ and the throughput $T$ in the asymptotic settings for the optimal $\beta \approx 3.1$




- Asymptotically optimal way to maximize throughput:
  - End the contention when the throughput starts to drop

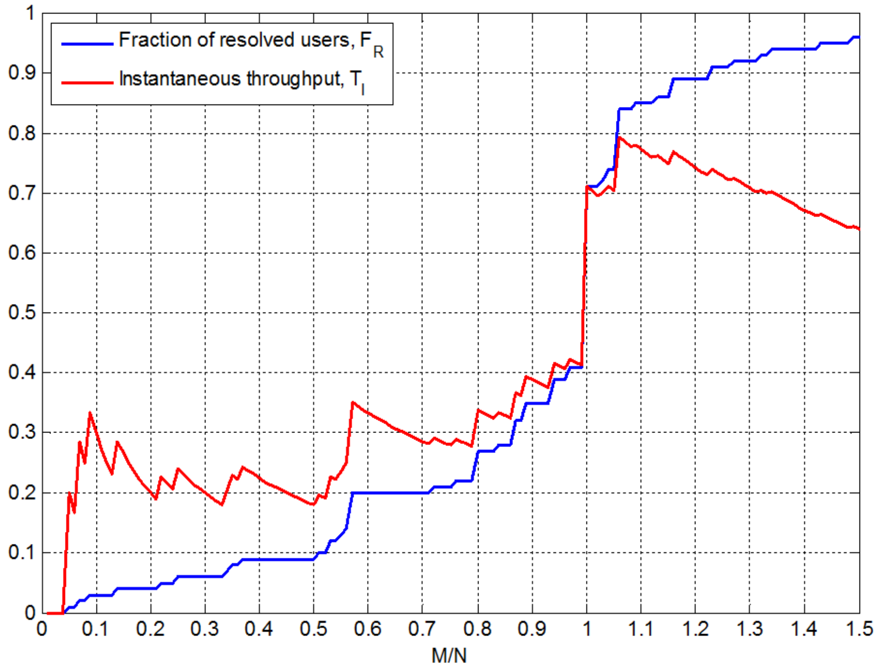
# Frameless ALOHA: Optimizing the stopping criterion

- An example of a typical run of frameless ALOHA in terms of:
  - fraction of resolved users

$$F_R = \frac{N_R}{N}$$

  - instantaneous throughput

$$T_R = \frac{N_R}{M}$$




genie-aided stopping criterion: stop when T is maximal

heuristic stopping criterion: fraction of resolved users

# Frameless ALOHA: Optimizing the stopping criterion

- Heuristic termination criterion:
- Stop the contention when:
  - $F_R \geq V$, or
  - $T_I = 1$
- The highest reported non-asymptotic throughputs so far

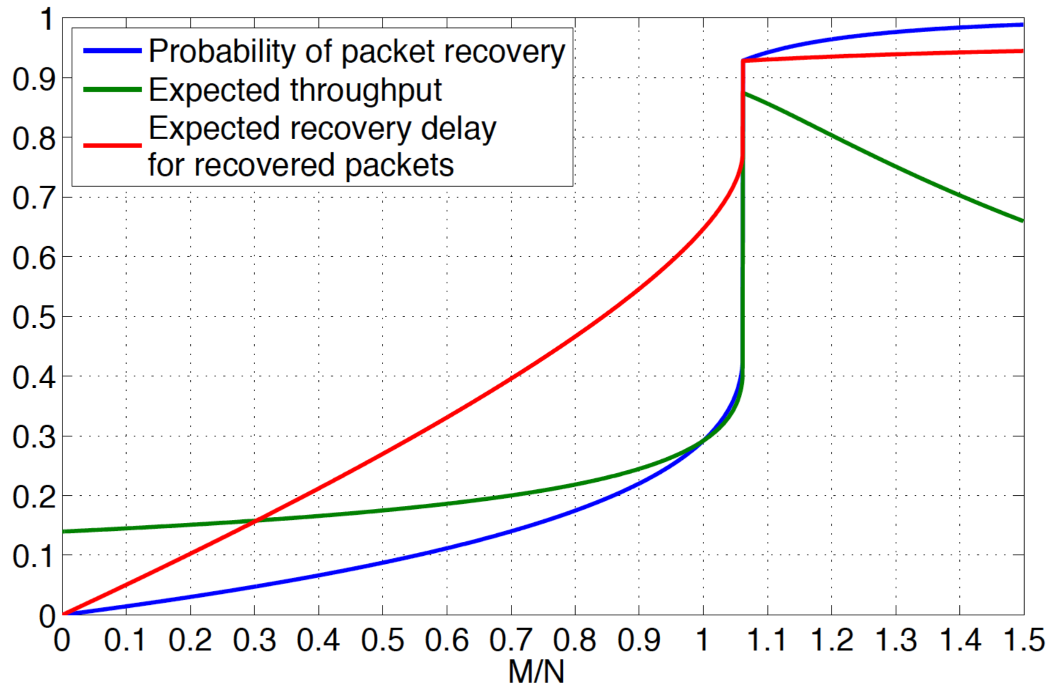
| $N$ | 50 | 100 | 500 | 1000 |
|---|---|---|---|---|
| $T_{GA}$ | 0.83 | 0.84 | 0.88 | 0.88 |
| $T$ | 0.82 | 0.84 | 0.87 | 0.88 |
| $F_R$ | 0.75 | 0.76 | 0.76 | 0.76 |
| $M/N$ | 0.97 | 0.95 | 0.9 | 0.9 |
| $\beta$ | 2.68 | 2.83 | 2.99 | 3.03 |
| $V$ | 0.83 | 0.87 | 0.88 | 0.89 |

# Frameless ALOHA: Average delay

- The frameless structure provides an elegant framework
  to compute the average delay of the resolved users
- Average delay as a function of the total number of contention slots M

$$D(M) = \sum_{m=1}^{M} (1 - P_R(m)/P_R(M))$$

- Observations
  - Average delay shifted towards the end of the contention period
  - Most of the users get resolved close to the end
  - Typical for the iterative belief-propagation
  - NB: we have not optimized the protocol for
    delay minimization



# Noise–induced errors

- Received signal in a slot when the noise is plugged in:

$$Y = \sum_k X_k + Z$$

- Theorem:

The throughput of the “noisy” frameless ALOHA with perfect SIC is:

$$T_n = T \cdot P_D$$

where $T$ is the throughput of the noiseless frameless ALOHA with perfect SIC and $P_D$ is the probability that a singleton slot is useful.

# Frameless ALOHA: Estimating the number of contending users

- C. Stefanovic, K. F. Trilingsgaard, N. K. Pratas, P. Popovski, "Joint Estimation and Contention-Resolution Protocol for Wireless Random Access", IEEE ICC 2013.

- Slot access probability:

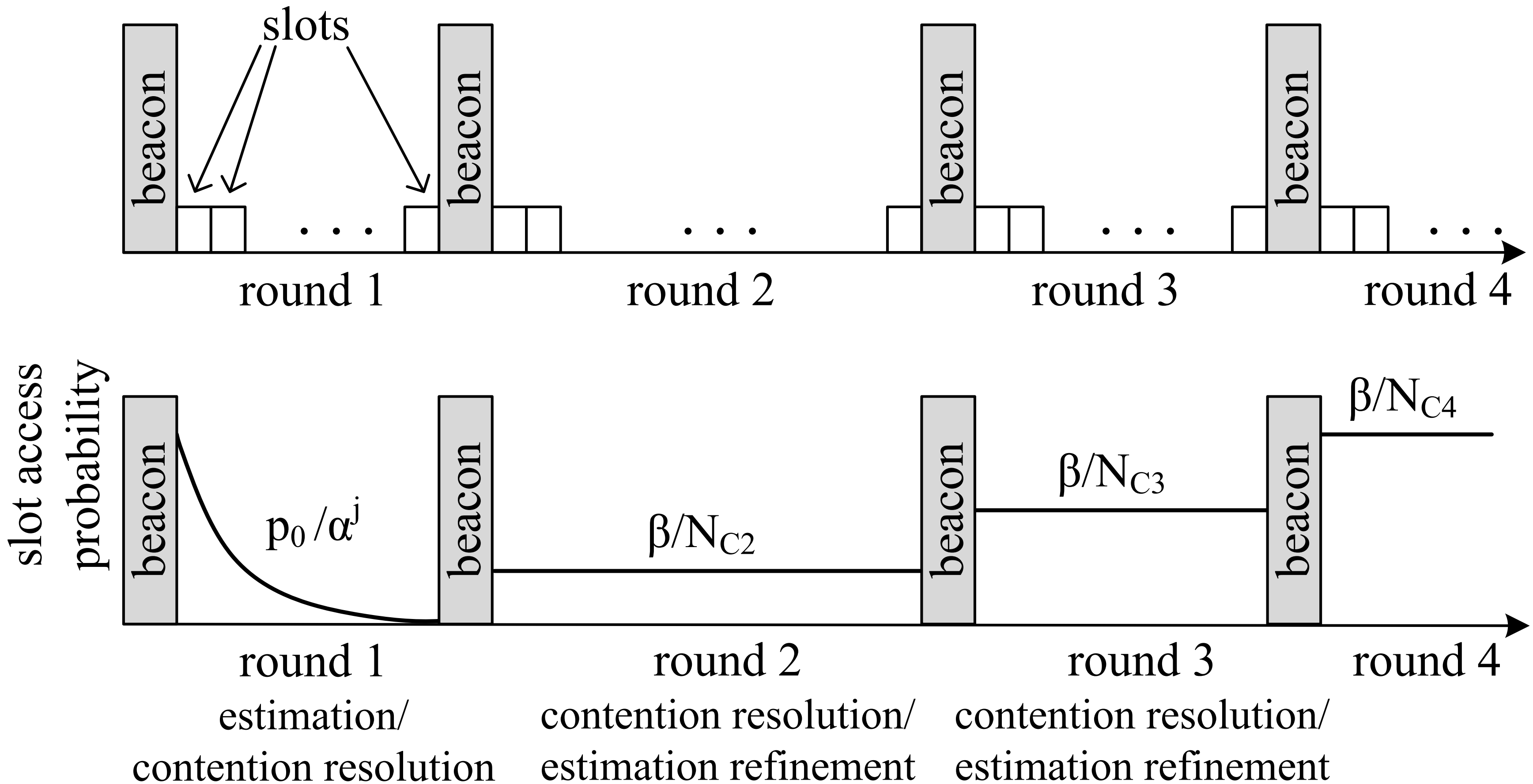
$$p_{\mathrm{A}} = \frac{\beta}{N}$$

- Termination criterion:

$$F_R = \frac{N_R}{N} \geq V$$

Requires estimation of the number of contending users $N$

# Frameless ALOHA: Estimating the number of contending users



# Estimation algorithm

- Notation:
  - $\hat{N}$ - estimation of $N$, $N_{Ci}$ - number of users contending in the i-th round
  - $s_{ij}$ - j-th slot of the i-th slot round, $|s_{ij}|$ - corresponding degree
  - $p_{ij}$ - corresponding slot-access probability

- Slots (i.e., observations) can be idle, single or collision slots
- Pmf of the observations, given (unknown) $N_{Ci}$ is:

$$f(s_{ij}|n) = \begin{cases} (1-p_{ij})^{n_{Ci}} & \text{if } |s_{ij}| = 0, \\ n_{Ci}p_{ij}(1-p_{ij})^{n_{Ci}-1} & \text{if } |s_{ij}| = 1, \\ 1-(1-p_{ij})^{n_{Ci}} - & \\ -n_{Ci}p_{ij}(1-p_{ij})^{n_{Ci}-1} & \text{if } |s_{ij}| > 1. \end{cases}$$

# Estimation algorithm

- MLE approach:

$$\hat{N} = \arg\max_n \prod_{i,j} f(s_{ij}|n) = \arg\max_n \sum_{i,j} \ln f(s_{ij}|n)$$

- Slots are independent:

$$\sum_{s_{ij} \in \mathcal{O}_0 \cup \mathcal{O}_1} \ln(1 - p_{ij}) + \sum_{s_{ij} \in \mathcal{O}_1} \frac{1}{n_{Ci}} +$$
$$\sum_{s_{ij} \in \mathcal{O}_C} \frac{(1-p_{ij})^{n_{Ci}}[1 + \ln(1-p_{ij})(\frac{1}{p_{ij}} + n_{Ci} - 1)]}{1 - \frac{1}{p_{ij}} + (1-p_{ij})^{n_{Ci}}(\frac{1}{p_{ij}} + n_{Ci} - 1)} = 0$$

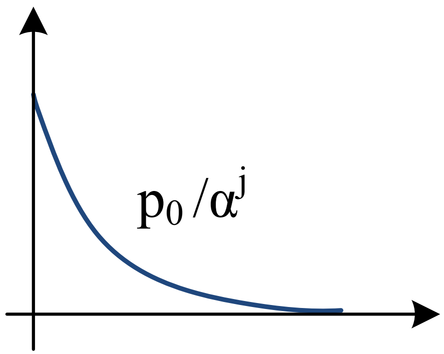
  – Can be efficiently solved using some root-finding method, e.g. Brent’s method

# Estimation algorithm

- End the estimation round when $K$ successive idle slots are observed
- Hard to deal with analytically
- Simulations show that the estimator is unbiased and its output has a Gaussian pdf

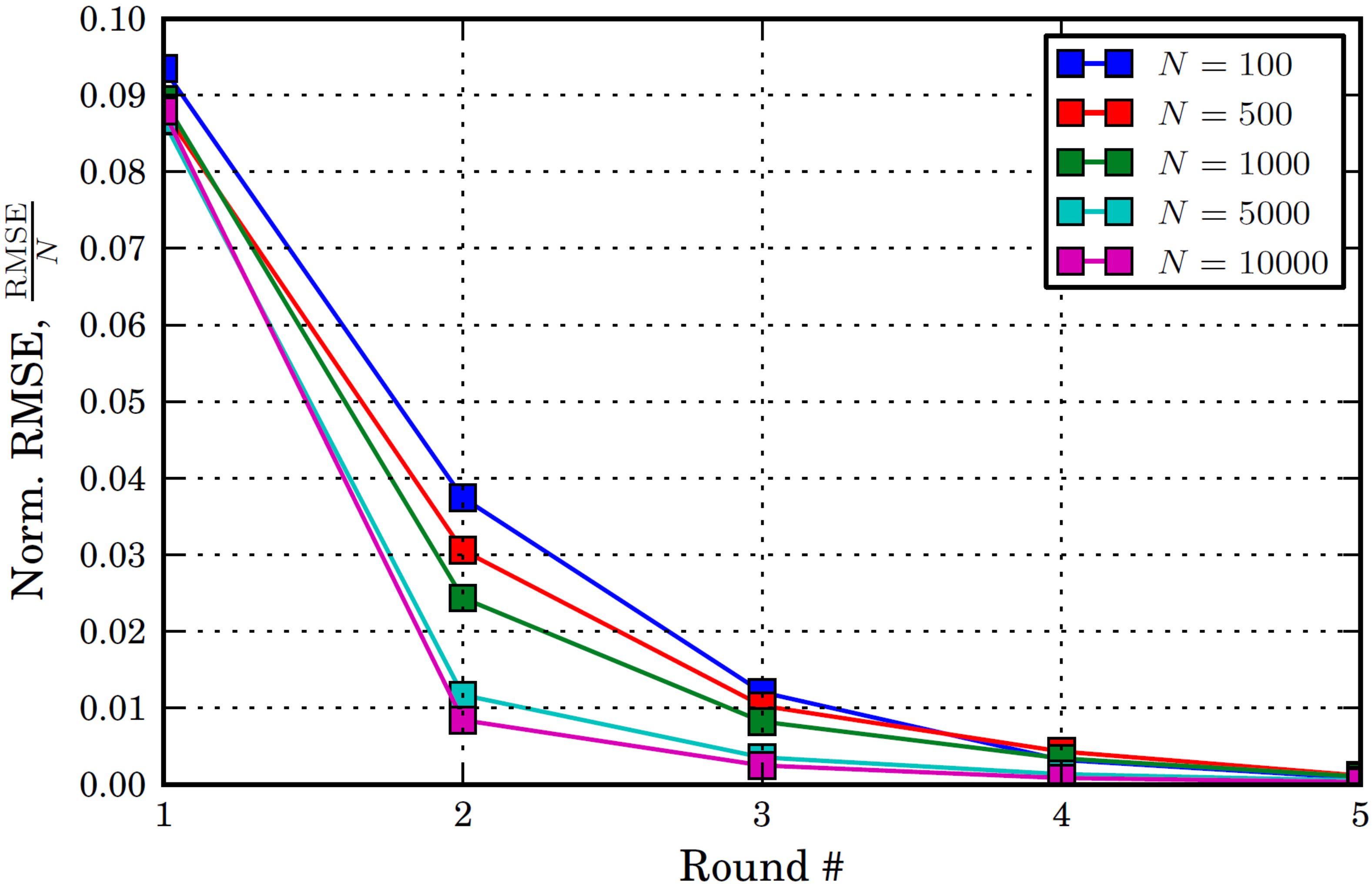
- Slot access probability in the estimation round changes as:

$$p_{1j} = \frac{p_0}{\alpha^j}$$

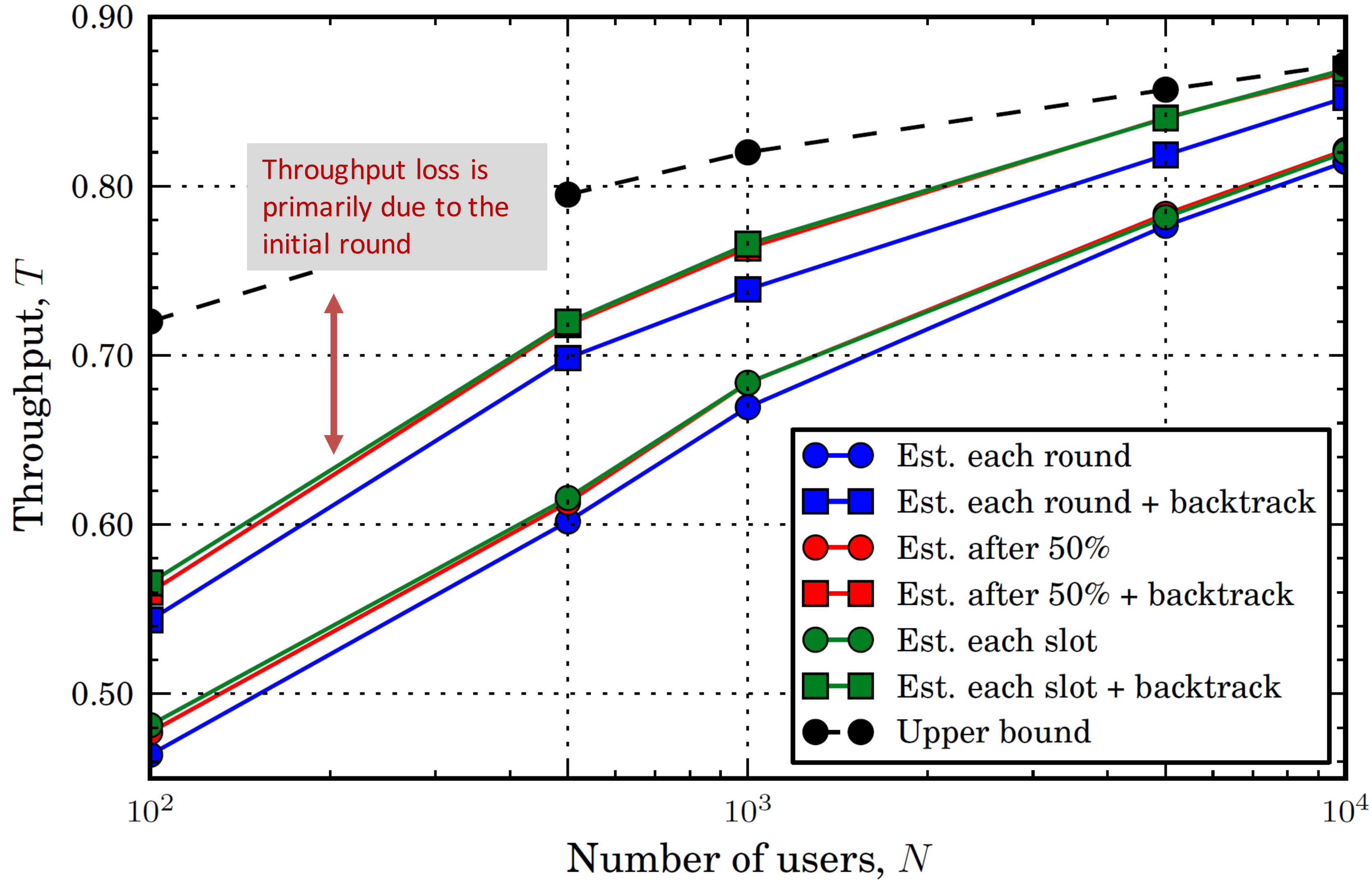



- RMSE depends on $N$, $K$, $p_0$ and $\alpha$

# Estimation algorithm: Root-mean square error



# Frameless ALOHA: Throughput with the estimation included



# Capture effect

- So far, we assumed that the collision slots can be exploited only when they become singleton slots through successive cancellation of the already resolved transmissions
  - I.e., through inter-slot SIC

- However, in practice, collision slots may be already exploitable due to the capture effect
  - Capture effect often occurs in wireless communications

- Typical model:
  - Transmission of user $u_i$ is captured in slot if the following condition is satisfied:

$$\frac{P_i}{P_n + \sum_j P_j} \geq b$$

Received power of $u_i$

Capture threshold

Noise power

Received power of interfering users

# Capture effect

- Capture effect implicitly assumes unequal received powers of the user transmissions:

$$Y = \sum_k h_k X_k + Z$$

- Example:

$$Y = 10X_1 + X_2 + Z$$

- There is a chance that $u_1$ captures the slot over $u_2$ and the noise

# Capture effect in coded slotted ALOHA

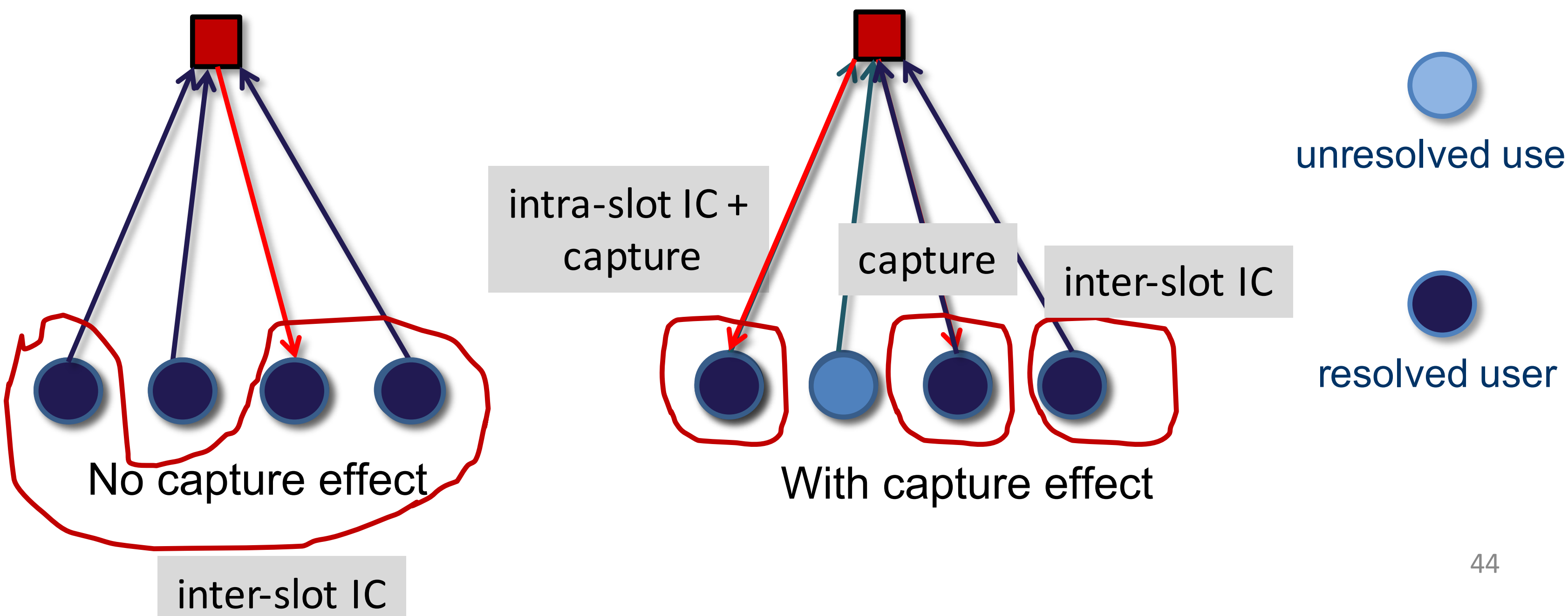
- Example:

$$Y = 10X_1 + X_2 + Z$$

- If $u_1$ captures the slot, then it is decoded and removed (perfectly) which yields:

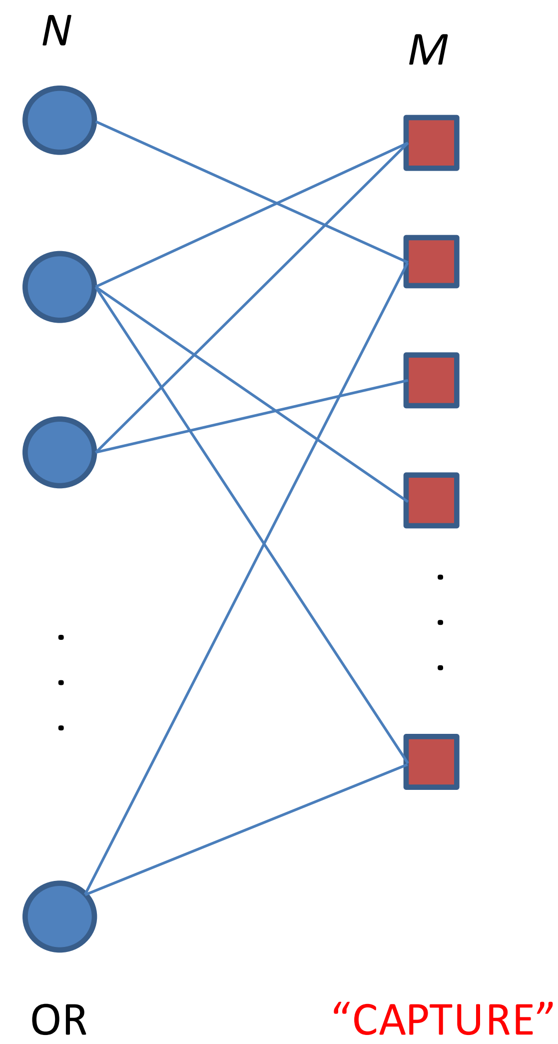
$$Y = X_2 + Z$$

  - so, there is a probability that $u_2$ can capture the slot via inter-slot IC

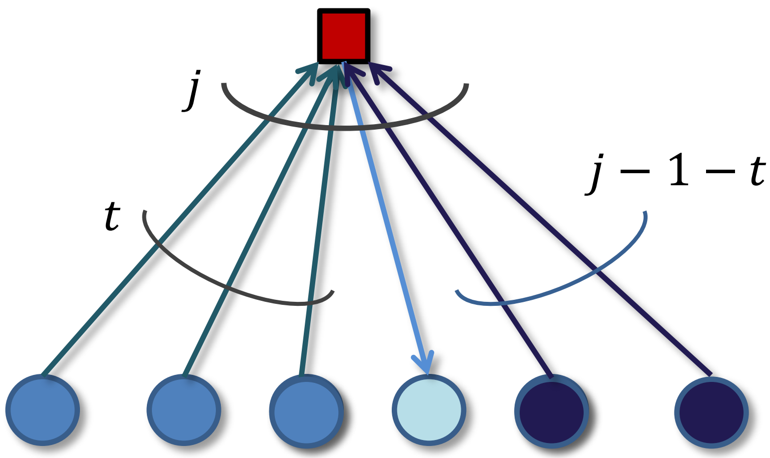


# Capture effect and and-or tree evaluation

- C. Stefanovic, M. Momoda, P. Popovski, "Exploiting Capture Effect in Frameless ALOHA for Massive Wireless Random Access", IEEE WCNC 2014.
- "CAPTURE" instead of AND operation
- Very hard to analyze in the general case
- Simplifying assumptions:
  - User channels are IID
  - Expected received powers are the same for all users
  - Interference cancellation is perfect



# Capture effect and and-or tree evaluation




- Narrowband system, valid for typical M2M scenarios:
  - Capture threshold $b \le 1$
- Rayleigh fading scenario
  - pdf of SNR user $u_i$ at the reception point:

$$p_{X_i}(x) = \frac{1}{\gamma} e^{-\frac{x}{\gamma}}, x \ge 0$$

    - $\gamma$ – the same expected SNR for every user

- AND operation becomes:

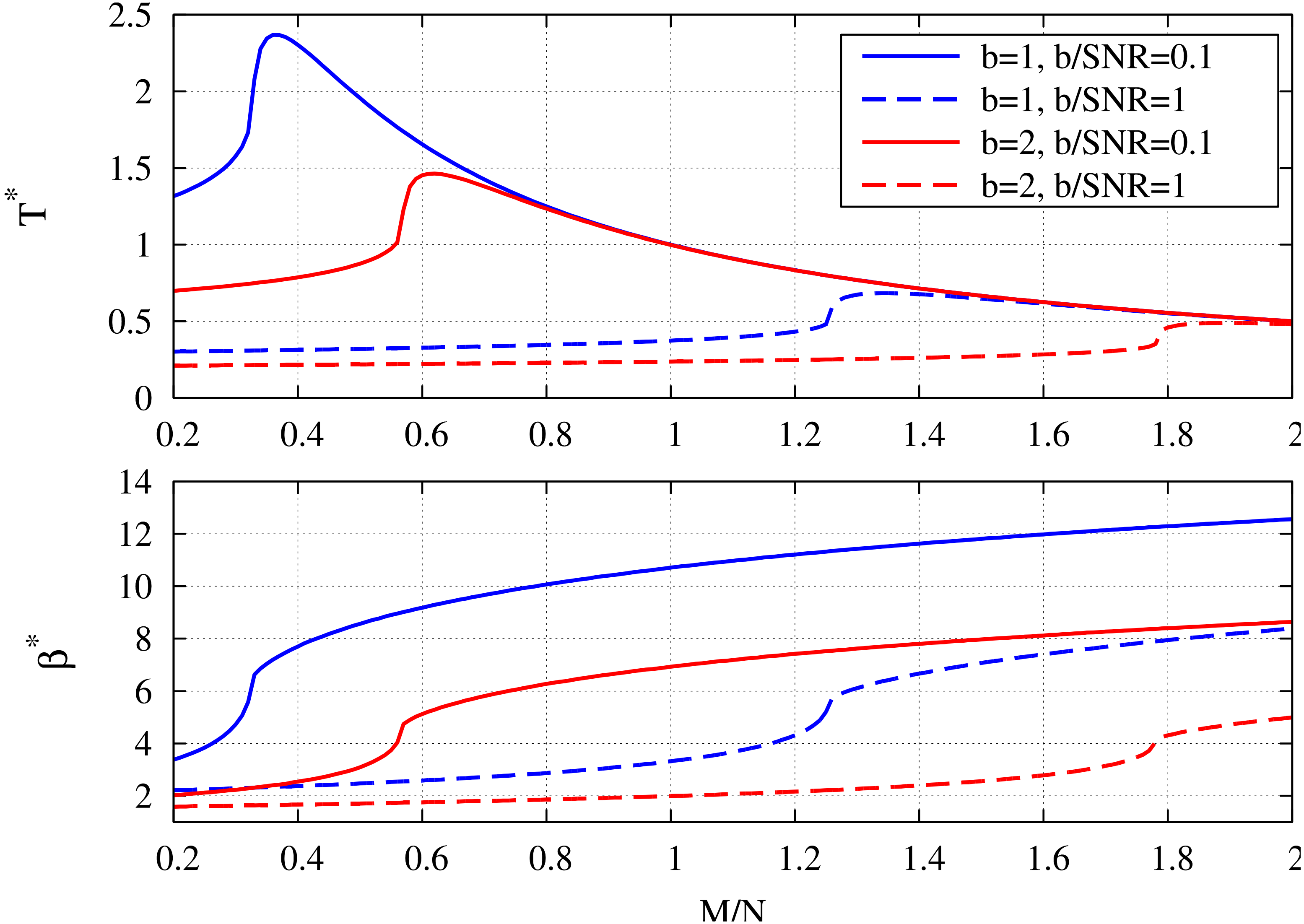
$$p = \sum_j \omega_j \sum_{t=0}^{j-1} \pi_t \binom{j-1}{t} q^t (1-q)^{j-1-t}$$

Probability that user captures slot when t interfering users remain

- Capture probability is:

$$\pi_t = \sum_{h=1}^{t+1} \frac{e^{\frac{1-(1+b)^h}{\gamma}}}{(1+b)^{\left(t-\frac{h-1}{2}\right)h}}, \qquad t \ge 0$$

# Frameless ALOHA with capture: Asymptotic analysis



# Frameless ALOHA with capture: Non-asymptotic results

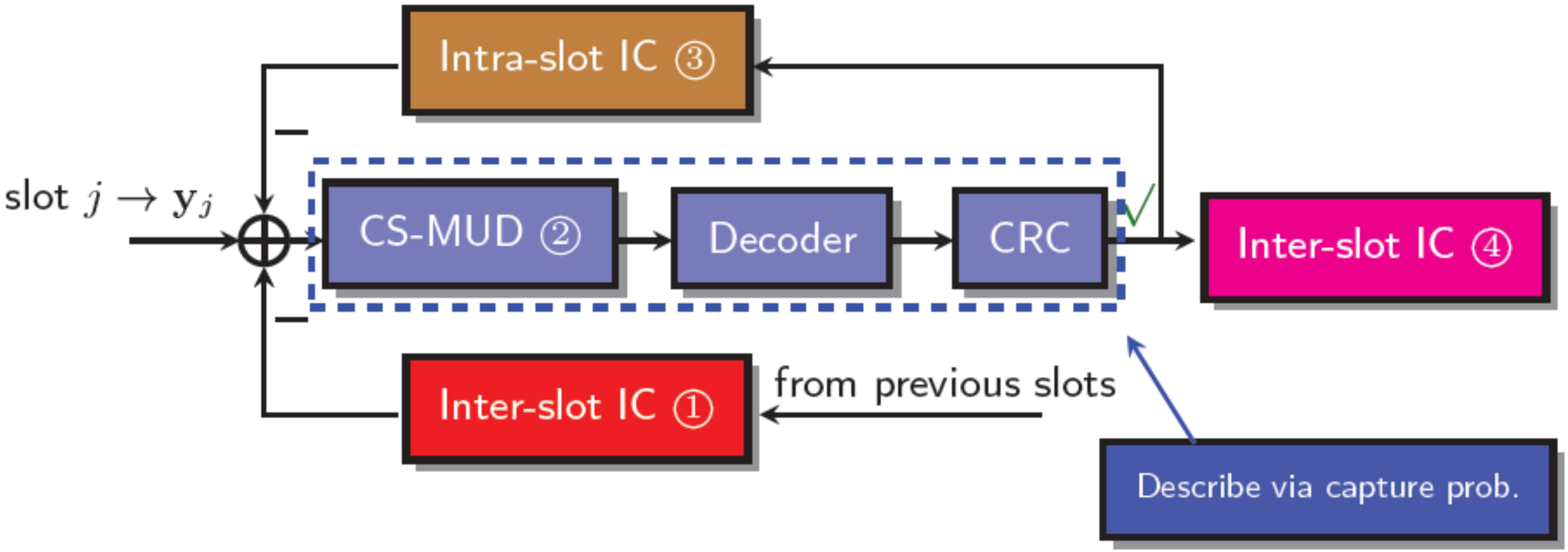
| $N$ | $b$ | 1 | | 2 | | no capture and no noise |
|---|---|---|---|---|---|---|
| | $b/\gamma$ | 0.1 | 1 | 0.1 | 1 | |
| 100 | $T$ | 1.92 | 0.41 | 1.21 | 0.31 | 0.8 |
| | $G$ | 6.14 | 2.23 | 4.53 | 1.55 | 2.89 |
| 1000 | $T$ | 2.13 | 0.42 | 1.33 | 0.32 | 0.86 |
| | $G$ | 6.91 | 2.38 | 5.1 | 2.15 | 3.04 |
| $\infty$ | $T$ | 2.37 | 0.68 | 1.46 | 0.49 | 0.88 |
| | $G$ | 7.2 | 6.37 | 5.29 | 4.69 | 3.12 |

# Frameless ALOHA with capture

- For high SNR, i.e., low $b/SNR$, substantially higher throughputs can be achieved
  - Throughput is well over 1!
  - The throughput decreases as $b$ increases
- For low SNR, i.e., high $b/SNR$, the achievable throughputs drop
  - This is due to the impact of noise
- Target slot degrees $G$ are higher than in the case without capture effect
  - I.e., the capture effect favors collisions

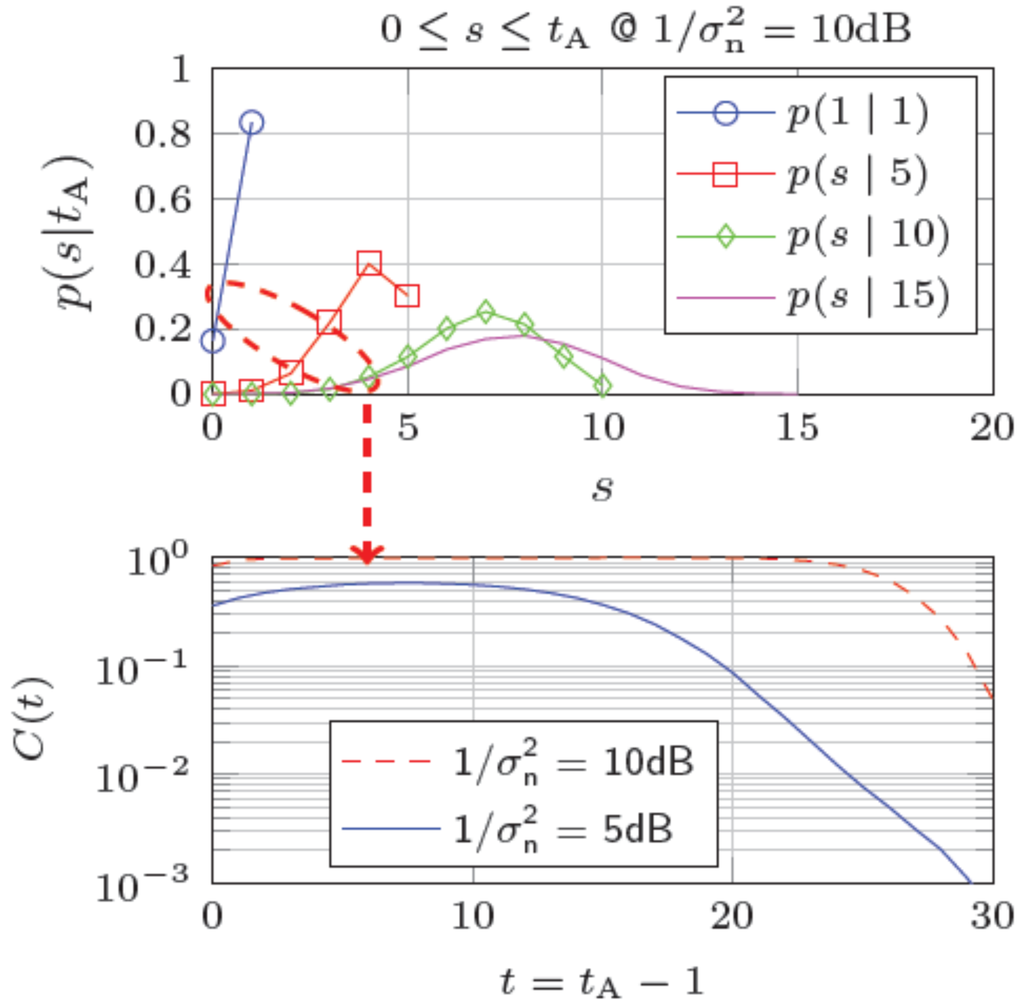
# Frameless ALOHA with capture and with compressive sensing receiver

- Y. Ji, C. Stefanovic, C. Bockelmann, A. Dekorsy, Petar Popovski, "Characterization of Coded Random Access with Compressive Sensing based Multi-User Detection", IEEE Globecom 2014
- Compressive sensing receiver = Multi-user detection (MUD)

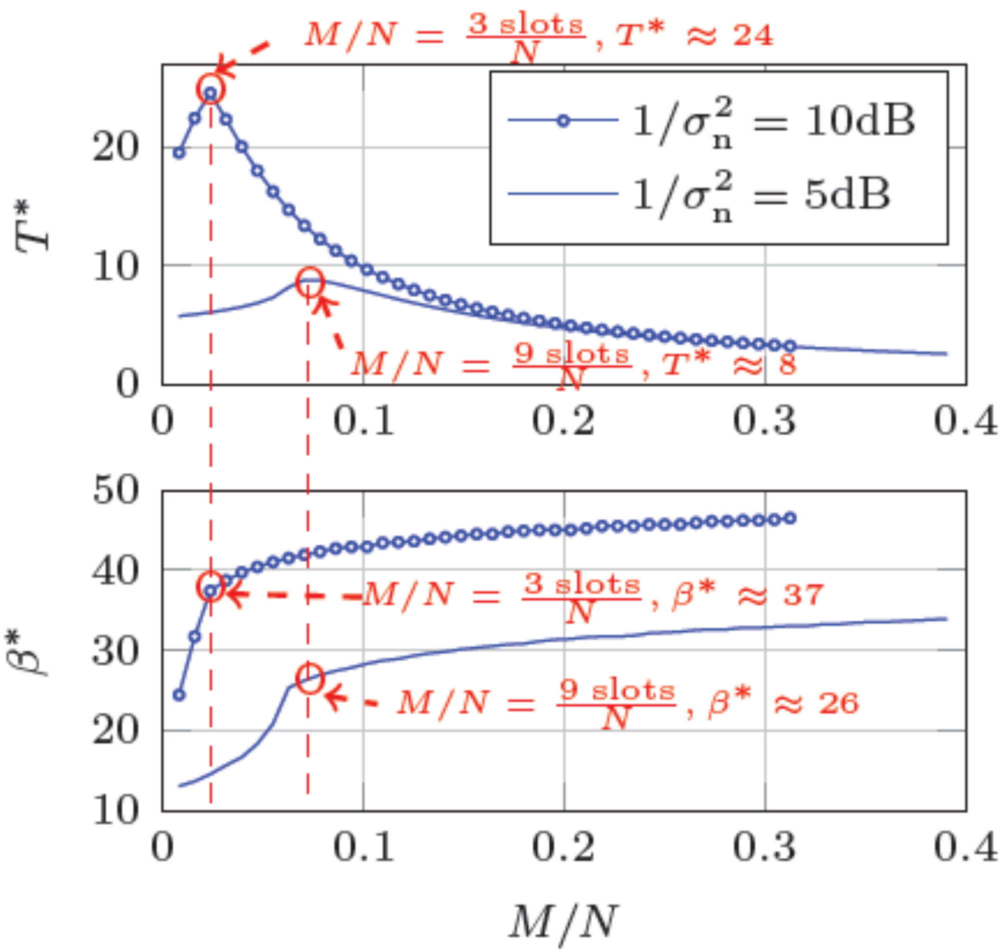


# Frameless ALOHA with capture and with compressive sensing receiver

- Evaluation setup:
  - Multi-User uplink CDMA system
  - No. of nodes: N = 128
  - Spreading sequence Ns = 32
  - L = 104 symbols per frame & BPSK
  - Conv. code $R_c = 0.5$ & constraint length $l_c = 3$
  - Freq. selective Rayleigh fading
  - Perfect channel knowledge at Base Station
  - Group Orthogonal Matching Pursuit as CS-MUD algorithm



# Frameless ALOHA with capture and with compressive sensing receiver




- Observations:
  - 10dB: less slots -> mainly intra-slot IC
  - 5dB: more slots -> inter- and intra-slot IC

# Frameless ALOHA with capture and imperfect SIC

- Model:
  - Transmission of user $u_i$ is captured in slot if the following condition is satisfied:

$$\frac{P_i}{P_n + \sum_j P_j + \sum_k Q_k} \geq b$$

Received power of $u_i$

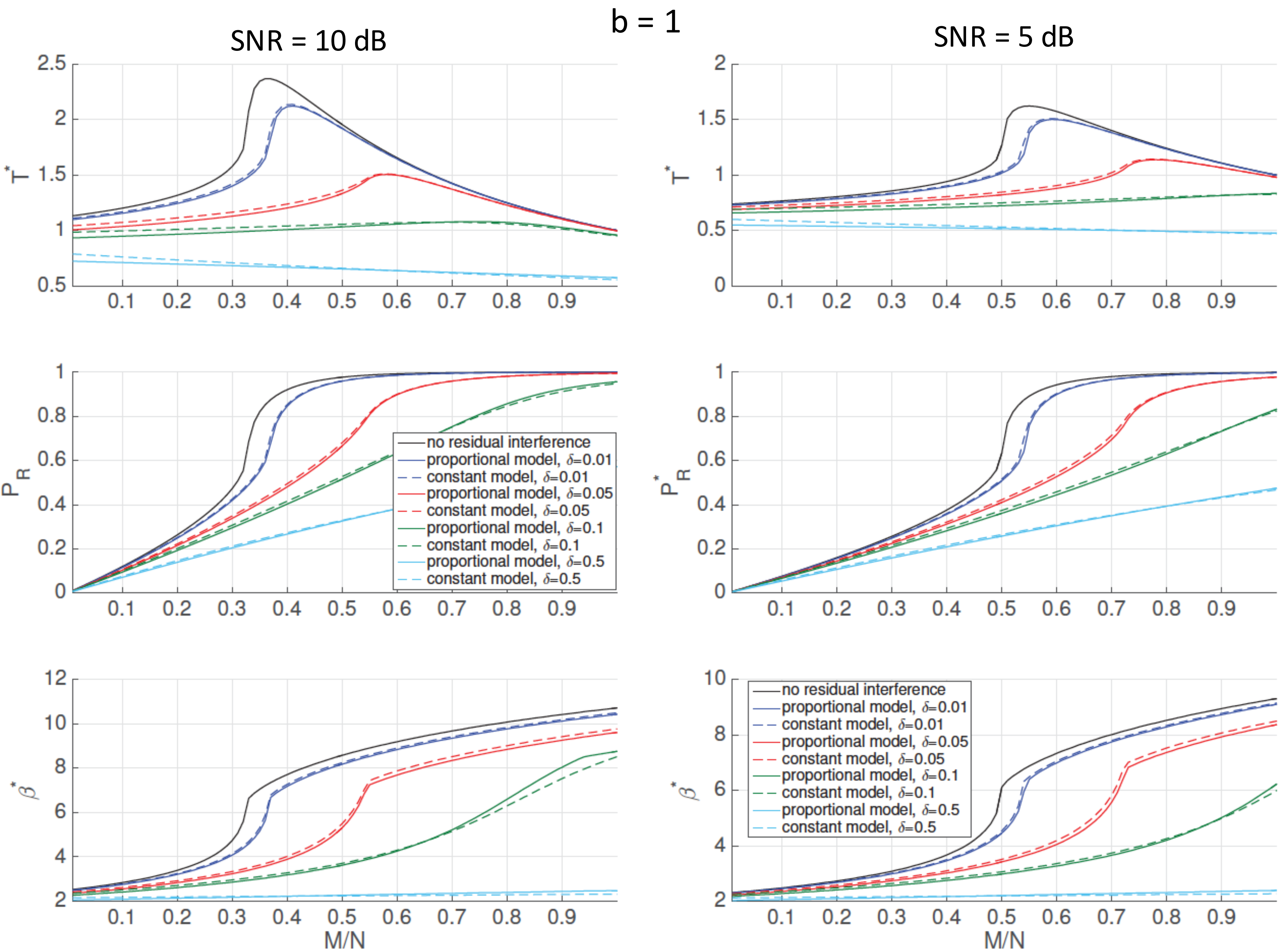
Capture threshold

Noise power

Residual power of cancelled users

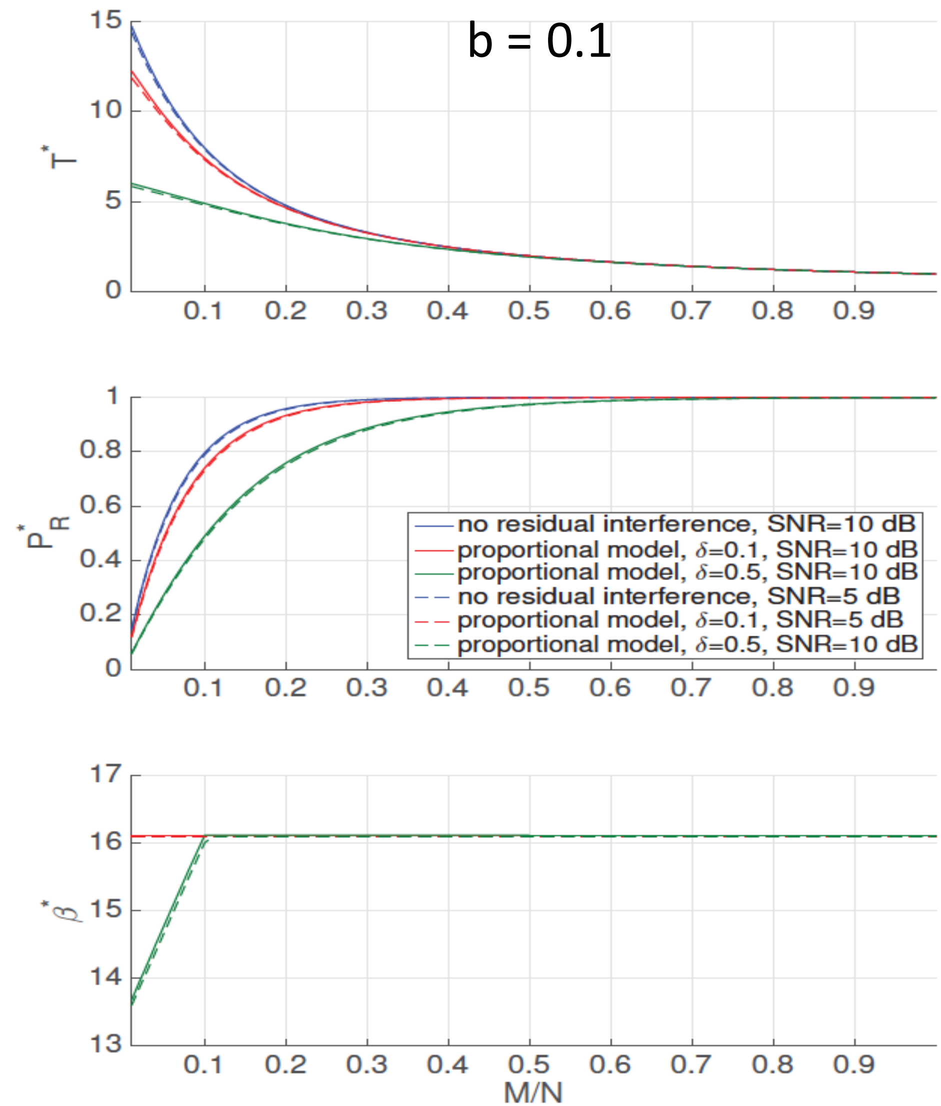
Received power of remaining interfering users

- Slot degree $|s| = j + k + 1$
- As user resolution progresses, users interfering in the same slot "move" from $\sum_j P_j$ to $\sum_k Q_k$


b = 1
SNR = 10 dB
SNR = 5 dB
T*
P*R
β*
M/N
no residual interference
proportional model, δ=0.01
constant model, δ=0.01
proportional model, δ=0.05
constant model, δ=0.05
proportional model, δ=0.1
constant model, δ=0.1
proportional model, δ=0.5
constant model, δ=0.5

# Frameless ALOHA with capture and imperfect SIC in wideband scenario

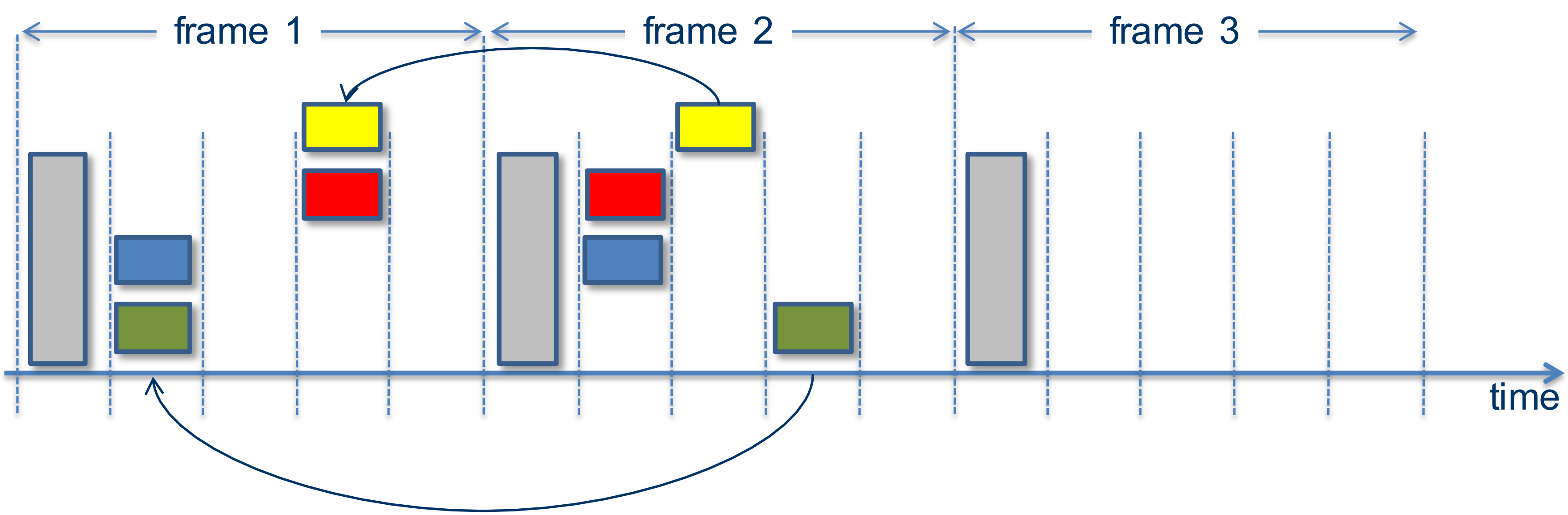
- So far, we assumed a narrowband scenario
  - I.e., capture threshold is:
    $$b \geq 1$$
  - At any moment, only a single user can capture the slot
- Wideband scenario:
  $$b < 1$$
  - Multiple users can capture the slot simultaneously



# Applying coded random access to existing protocols

- E. Paolini, C. Stefanovic, G. Liva, P. Popovski, "Coded Random Access: Applying Codes on Graphs to Design Random Access Protocols", IEEE Communications Magazine, Jun. 2015

- Coded random access virtualizes multiple frames in a super-frame and runs the SIC algorithm




Requires upgrade in the Base Station,
but no changes to the transmission format at the devices

**FIN**