Compressive Coded Random Access for Massive MTC Traffic in 5G Systems

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***Abstract*—Massive MTC support is an important future market segment, but not yet efficiently supported in cellular systems. In this paper we follow-up on recent concepts combining advanced MAC protocols with Compressed Sensing (CS) based multiuser detection. Specifically, we introduce a concept for sparse joint activity, channel and data detection in the context of the Coded ALOHA (FDMA) protocol. We will argue that a simple sparse activity and data detection is not sufficient (as many papers do) because control resources are in the order of the data. In addition, we will improve on the performance of such protocols in terms of the reduction of resources required for the user activity, channel estimation and data detection. We will mathematically analyze the system accordingly and provide expressions for the capture probabilities of the underlying sparse multiuser detector. Finally, we will provide CS algorithms for the joint estimation scheme and evaluate its performance.**

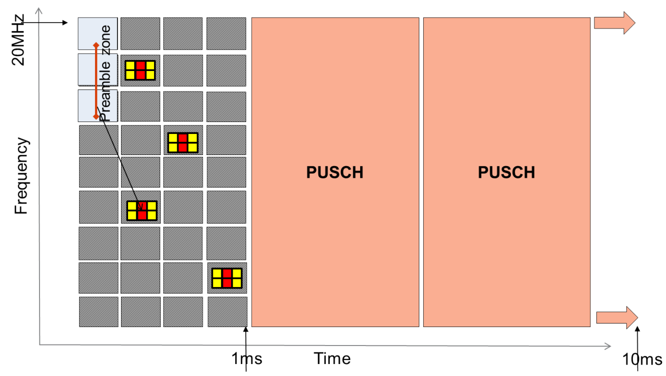

Fig. 1. Random access concepts: 5G standard approach with separated preamble section for ”activity” detection and respective pointer to data packet.

I. Introduction

The Internet of Things (IoT) is a most promising 5G market segment and in the focus of all key players in the ICT domain. Even pessimistic forecasts predict several billions of connected devices. Major proliferation of the IoT will be naturally in the 5G wireless domain. Currently, IoT market is mainly served by short range capillary wireless technologies such as Bluetooth LE, ZigBee, and WiFi and proprietary (clean slate) low power wide area technologies such as SIGFOX, LoRA etc. There is only small share for cellular and there is clearly a need to act fast in this direction.

IoT requires support of scalable massive machine-type communication (MTC), which is essentially a sporadic traffic pattern generated by devices operating under tight (resource) constraints such as low cost, battery lifetime, computation capability etc. Such messages have typically very unfavorable control/data signaling ratio; recent proposals suggest 5G “one-shot” random access concepts where devices wake up and send data right away with no coordination whatsoever [1], [2]. The concept is depicted in Fig. 1. While this concept is quite appealing it comes with significant challenges:

(i) Temporal asynchronous access among different resources; spectral asynchronous access due to low-cost terminals; definition of shorter TTIs and more granularity in allocating the physical resource blocks. *This is the waveform challenge* [1].
(ii) Relationship between the data and the control data (metadata); control signaling possibly in the order of data; per user resource control signaling becomes inefficient. *This is the metadata challenge* [1].
(iii) Throughput severely degraded due to collisions in random access unless successive cancellation is applied. *This is the throughput challenge* [3].

The challenges are depicted in Fig. 2. In this paper, we address the throughput challenge and follow-up on recent concepts combining advanced MAC protocols with Compressed Sensing (CS) based multiuser detection. Specifically, we introduce a concept for sparse joint activity, channel and data detection in the context of the Coded ALOHA (FDMA) protocol which we call *Compressive Coded Random Access* (CCRA) extending the work in [4], [5], [6]. We will argue that a simple sparse activity and data detection is not sufficient (as many papers do) because control resources are in the order of the data. In addition, we will improve on the performance of such protocols in terms of the reduction of resources required for the user activity, channel estimation and data detection. We will mathematically analyze the system accordingly and provide expressions for the capture probabilities of the underlying sparse multiuser detector. Finally, we will provide CS

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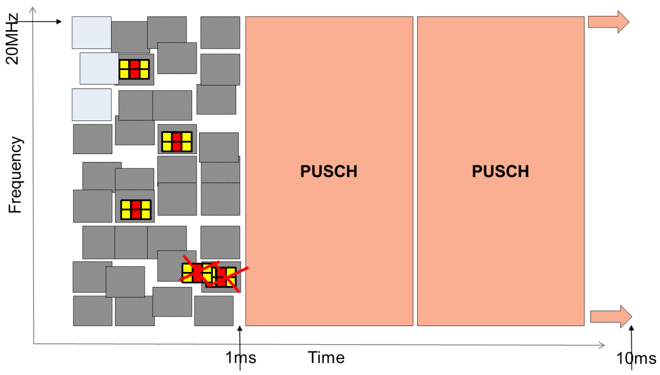

Fig. 2. Random access concepts: Traditonal approach.

algorithms for the joint estimation scheme and evaluate its performance.

Notations. $\|x\|_{\ell_q} = (\sum_i |x_i|^q)^{1/q}$ is the usual notion of ℓ_q-norms and $\|x\| := \|x\|_{\ell_2}$. We denote with $\text{supp}(x) := \{i : x_i := \langle e_i, x\rangle \neq 0\}$ the support of x in a given fixed (here canonical) basis $\{e_i\}_{i=1}^n$. The size of its support is denoted as $\|x\|_{\ell_0} := |\text{supp}(x)|$. W is the (unitary) Fourier matrix with elements $(W)_{ij} = n^{-\frac{1}{2}} e^{-\sqrt{-1}2\pi ij/n}$ for $k, l = 0 \ldots n-1$, hence, $W^{-1} = W^*$ where W^* is the adjoint of W. We use $\hat{x} = Wx$ to denote Fourier transforms and $\odot$ means point-wise product. I_n is the identity matrix in $\mathbb{C}^n$, $\text{diag}(x)$ is some arbitrary diagonal matrix with $x \in \mathbb{C}^n$ on its diagonal.

II. CCRA MODEL

For simplicity assume one time slot only and n OFDM subcarriers. This is easily generalized to the case where there are multiple time slots, notably, within the coherence time so that channels are constant over these slots. Let $p_u \in \mathbb{C}^n$ be some signature from a given set $\mathcal{P} \subset \mathbb{C}^n$ and $x_u \in \mathcal{X}^n$ be an unknown (uncoded) data sequence from the modulation alphabet $\mathcal{X}^n$ both for the u-th user with $u \in \{1, ..., U\}$ and U is the (fixed) maximum set of users in the systems. Note that in our system n is a very large number, e.g. 24k. Due to the random zero-mean nature of x_u we have $\frac{1}{n}E\|p_u + x_u\|^2 = 1$, i.e. the total (normalized) transmit power is unity. Provided user u is active, we set:

$$\alpha := \frac{1}{n}\|p_u\|^2 \quad \text{and} \quad \alpha' := 1 - \alpha = \frac{1}{n}E\|x_u\|^2$$

Hence, the control signalling fraction of the power is α. If a user is not active then we set both $p_u = x_u = 0$, i.e. either a user is active and seeks to transmit data or it is inactive. Let $h_u \in \mathbb{C}^s$ denotes the sampled channel impulse response (CIR) where $s \ll n$ is the length of the cyclic prefix. The most important assumptions in this paper are:

(i) Bounded support of h_u, i.e. $\text{supp}(h_u) \subseteq [0, \ldots, s-1]$ due to the cyclic prefix
(ii) Sparsity of h_u within $\text{supp}(h_u)$, i.e. $\|h_u\|_{l_0} \leq k_1$
(iii) Sparse user activity, i.e. k_2 users out of U in total are actually active.

Define $k := k_1 k_2$.

Let $[h, 0] \in \mathbb{C}^n$ denote the zero-padded CIR. The received signal is then:

$$\begin{aligned} y &= \sum_{u=0}^{U-1} \text{circ}([h_u, 0])(p_u + x_u) + e \\ y_{\mathcal{B}} &= \Phi_{\mathcal{B}} y \end{aligned}$$

Here, $\text{circ}([h_u, 0]) \in \mathbb{C}^n$ is the circulant matrix with $[h_u, 0]$ in its first column. $\Phi_{\mathcal{B}}$ denotes some measurement matrix (to be specified later on) typically referring to a frequency window $\mathcal{B}$ of size $m := |\mathcal{B}|$. All performance indicators depend strongly on the number of subcarriers in $\mathcal{B}$ (control) and $\mathcal{B}^C$ (data). The goal is clearly a small observation window $\mathcal{B}$.

The AWGN is denoted as $e \in \mathbb{C}^n$ with $E(ee^*) = \sigma^2 I_n$. For circular convolutions we have $\text{circ}([h, 0])p = \sqrt{n} \cdot W^*(\hat{h} \odot \hat{p})$ so that:

$$\begin{aligned} y &= \sum_{u=1}^{U} W^* \left[(\sqrt{n}\hat{h}_u \odot (\hat{p}_u + \hat{x}_u) \right]) + \hat{e} \\ y_{\mathcal{B}} &= \Phi_{\mathcal{B}} y \end{aligned}$$

where e and $\hat{e}$ are statistically equivalent.

A. Control signaling model

For the CCRA scheme let us assume that users' preambles 'live' entirely in $\mathcal{B}$ while all data resides in $\mathcal{B}^C$, so that $\text{supp}(p_u) \subseteq \mathcal{B}\, \forall u$. We call this a common overloaded control channel [6]. Let $P_{\mathcal{B}} : \mathbb{C}^n \to \mathbb{C}^m$ be the corresponding projection matrix, i.e. the submatrix of I_n with rows in $\mathcal{B}$. For identifying which preamble is in the system we can consider $\hat{y}$ and use the frequencies in $\mathcal{B}$, i.e. $\Phi_{\mathcal{B}} = P_{\mathcal{B}} W$, so that:

$$y_{\mathcal{B}} := P_{\mathcal{B}} \sum_{u=1}^{U} \left[\sqrt{n}\hat{h}_u \odot (\hat{p}_u + \hat{x}_u) \right] + P_{\mathcal{B}}\hat{e}$$

For algorithmic solution, we can stack the users as:

$$\begin{aligned} y &= \sum_{u=1}^{U} \text{circ}(h_u)(p_u + x_u) + e \\ &= D(p)h + C(h)x + e \end{aligned}$$

where $D(p) := [\text{circ}(p_1), \ldots, \text{circ}(p_U)] \in \mathbb{C}^{n \times Un}$ and $C(h) := [\text{circ}([h_1, 0]), \ldots, \text{circ}([h_U, 0])] \in \mathbb{C}^{n \times Un}$ are the corresponding compound matrices, respectively $p = [p_1^T\ p_2^T\ ...p_U^T]^T$ und $h = [h_1^T\ h_2^T\ ...h_U^T]^T$ are the corresponding compound vectors. If we assume each user-channel vector h_u to be k_1-sparse and k_2 are active then h is k-sparse.

For joint user activity detection and channel estimation exploiting the sparsity we can use the standard basis pursuit denoising (BPDN) approach:

$$\hbar = \arg\min_h \|h\|_{\ell_1} \text{ s.t. } \|\Phi_{\mathcal{B}}\, D(p)h - y\|_{\ell_2} \leq \epsilon \tag{1}$$

Moreover, several greedy methods such as CoSAMP exists for sparse reconstruction. After running the algorithm in eqn. (1) the decision variables $\|\hbar_u\|_{\ell_2}^2\ \forall u$, are formed, indicating that if $\|\hbar_u\|_{\ell_2}^2 > \xi$ where $\xi > 0$ is some predefined threshold the user is considered active and its corresponding data is detected.

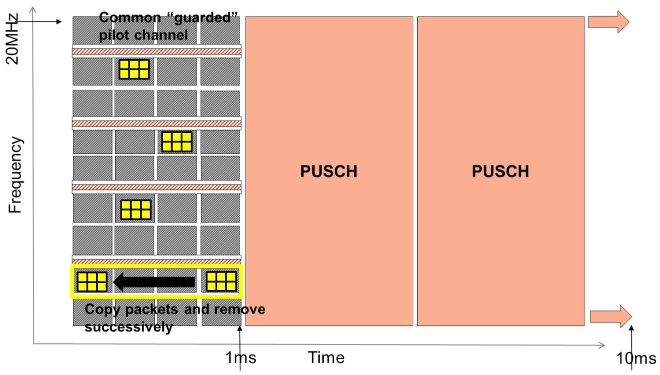

Fig. 3. Random access concepts: Approach.

B. Data signaling model

Since data resides only in $\mathcal{B}^C$ the entire bandwidth $\mathcal{B}^C$ can be divided into B frequency patterns. Each pattern is uniquely addressed by the preamble and indicates where the data and corresponding copies are placed. the scheme works as follows: if a user wants to transmit a small data portion, the pilot/data ratio α is fixed and a preamble is randomly selected from the entire set. The signature determines where (and how many of) the several copies in the B available frequency slots are placed which are processed in a specific way (see below). Such copies can greatly increase the utilization and capacity of the traditional ALOHA schemes. The principle is depicted in Fig. 3.

The random access algorithm can be seen as in instance of *coded slotted ALOHA* framework [3], tuned to incorporate the particularities of the physical layer addressed in the paper, as described in the previous section. Specifically, the random access algorithms assumes that:

- the users are active in multiple frequency slots, denoted simply as slots in further text,
- the activity pattern, i.e., the choice of the slots is random, according to a predefined distribution,
- every time a user is active, it sends a replica of packet, which contains data,

Obviously, due to the random nature of the choice of slots, the access point (i.e. the base station) observes idle slots (with no active user), singleton slots (with a single active user) and collision slots (with multiple active users). Using a compressive sensing receiver, the base station, decodes individual users from non-idle slots, removes (cancels) the replicas from the slots in which they occur (the knowledge of which is learned through signatures), and tries to decode new users from the slots from which replicas (i.e. interfering users) have been cancelled. In this way, due to the cancelling of replicas, the slots containing collisions that previously may have not been decodable, can become decodable. This process is executed in iterations, until there are no slots from which new users can be decoded. The above described operation can be represented via graph, see Fig. 4.

The iterative interference cancellation (IC) resembles iterative belief-propagation erasure decoding, allowing the use of the related theoretical concepts to analyze and design random access algorithms. However, the important differences have to be taken account, stemming from the nature of the physical layer operation:

(i) The received singleton slots are not always decodable, i.e. they are decodable with a certain probability, which depends on the received SNR, channel estimation etc.

(ii) The received collision slots may be decodable, depending on the multi-user detection capabilities. Further, the unbalance of received signal powers due to varying channels that users experience, may cause capture effect, where a subset of the collided users may be decoded as a result of a favorable SINR.

(iii) Cancellation of replicas, in general, is not ideal, due to imperfect channel estimation and/or channel variations among the slots where replicas occurred, operation of the physical layer etc., and leaves a residual interference power. This implies that, as the IC progresses, the residual interference accumulates in the affected slots, which may prevent further decoding of the remaining user packets.

Analytical modeling of the above is the main prerequisite to assess the performance of the random access algorithm, which in turn, allows for the design of the probability distribution that governs the choice slots, and which is typically optimized to maximize the throughput, i.e., the number of resolved packets per slot [3].

In the context of application of coded slotted ALOHA to compressive-sensing based physical layer, some preliminary work can be found in [4]. Here we extend the approach, by taking into account a more detailed operation of the physical layer, which incorporates channel estimation and imperfect interference cancellation, as detailed in Section III-B.

III. PERFORMANCE ANALYSIS

The performance analysis is split into activity detection/channel estimation and the data part, where coded random access is included.

A. User detection/channel estimation

In the data model we assume that fast fading effects are averaged out due to coding over subcarriers. Hence, user rates are ergodic and are calculated as expectations over the fading distributions. Achievable rates crucially depend on the receive powers (user position, slow fading effects), channel estimation errors and corresponding interference from colliding users then [6]. The relevant expressions under erroneous channel estimation will be provided below.

Suppose user u as well as colliding users $u(j) \in \mathcal{C}_u, j = 1, ...|\mathcal{C}|$, which are detected before in some singleton slot have been assigned subcarriers $i \in \mathcal{B}_u$. Due to the circular model each subcarrier has powers $E(|\hat{x}_{u,i}|^2) = 1 - \alpha$, $|\hat{p}_{u,i}|^2 = \alpha$ and $E(|\hat{e}_{u,i}|^2) = \sigma^2$. Denote the channel estimation error as $\hat{d}_{u,i} := \hat{\hbar}_{u,i} - \hat{h}_{u,i}$. Hence, the received signal is given by:

$$\hat{y}_{u,i} = (\sqrt{n}\hat{\hbar}_{u,i} + \hat{d}_{u,i})\hat{x}_{u,i} + \hat{e}_{u,i}$$

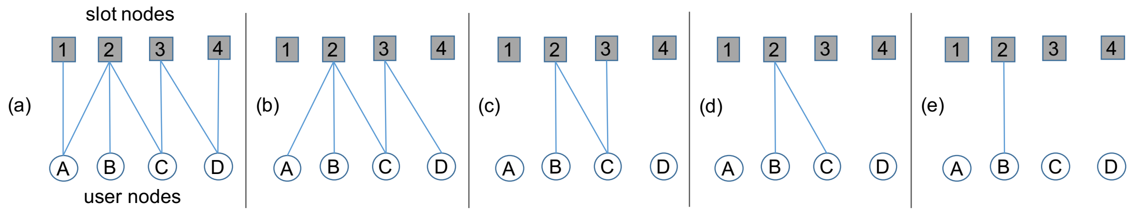

Fig. 4. Example of iterative IC decoding; it is assumed that only singleton slots are decodable, with probability 1. (a) The base stations decodes singleton slots 1 and 4, (b) obtaining packets of users A and D. (c) In the next step, the base station cancels replicas of decoded packets from slots 2 and 3, respectively, reducing slot 3 to a singleton slot. (d) The base station decodes packet of user C, and (e) cancels its replica from slot 2, which now becomes singleton.

for singleton slots and

$$\hat{y}_{u,i} = (\sqrt{n}\hat{\hbar}_{u,i} + \hat{d}_{u,i})\hat{x}_{u,i} + \sum_{j\in\mathcal{C}} \hat{d}_{u(j),i}\hat{x}_{u(j),i} + \hat{e}_{u,i}$$

for collision slots. Suppose further we have calculated the propability of not detecting an active user $P_{md}(\xi)$ ("missed detection"), and falsely detecting an inactive user $P_{fa}(\xi)$ ("false alarm"). Define $\bar{P}_{md}(\xi) := 1 - P_{md}(\xi)$ [6]. Let the channel impulse response be k-sparse and use BPDN as the channel estimate. Further, let $\Phi_{\mathcal{B}}, m = |\mathcal{B}|$, be a fixed measurement matrix with RIP constant $\delta_{2k} < \sqrt{2}-1$ and corresponding $c_1(\delta_{2k})$. The achievable rate $R(\alpha)$ per subcarrier for a particular user is lower bounded

- for singleton slots by:

$$R(\alpha) \geq E_{h|\{\|h\|>\xi\}}\left[\log\left(1+\frac{(1-\alpha)\,|h|^2}{\sigma^2}\right)\right]\bar{P}_{md}(\xi) - \log\left(1+\frac{(1-\alpha)\,c_1(\delta_{2k})^2 m}{\sigma^2\alpha n k_2}\right)$$

- and for collisions slots by:

$$R(\alpha) \geq E_{h|\{\|h\|>\xi\}}\left[\log\left(1+\frac{(1-\alpha)\,|h|^2}{\sigma^2}\right)\right]\bar{P}_{md}(\xi) - \log\left(1+\frac{(|\mathcal{C}|+1)\,(1-\alpha)\,c_1(\delta_{2k})^2 m}{\sigma^2\alpha n k_2}\right)$$

To prove we can extend the analysis in [6] in a straightforward manner. Note that the performance strongly depends on the scaling of $\frac{c_1(\delta_{2k})^2 m}{n k_2}$. From the CS literature upper and lower bounds are available (e.g. for CoSAMP see [7]), i.e. $c_1(\delta_{2k}) = 4\sqrt{1+\delta_{2k}}/1-(1+\sqrt{2})\delta_{2k}$ as well as bounds on the RIP constants δ_{2k} [8], but these bounds are rather loose so numerial simulations are still necessary.

B. Coded Slotted Aloha

The analysis of coded slotted ALOHA is typically based on the and-or tree evaluation [9]. It is assumed that the graph representation can be unfolded in a tree, see Fig. 5, on which two operations are performed in succession:

(i) decoding of user packets in slots, corresponding to (a generalized) "and" operation, cf. [10], [4],
(ii) removal of replicas, corresponding to "or" operation.

Both operations are probabilistically characterized, in terms of probability of *not* decoding a user in a slot, denoted as p_i, and *not* removing a replica q_i. The tree structure allows for their successive updates, as reflected in the subscripts of p_i and q_i. We also note that the analysis is asymptotic in nature, as in the non-asymptotic case, the graph representation contains loops, and the corresponding tree representation is only an approximation.

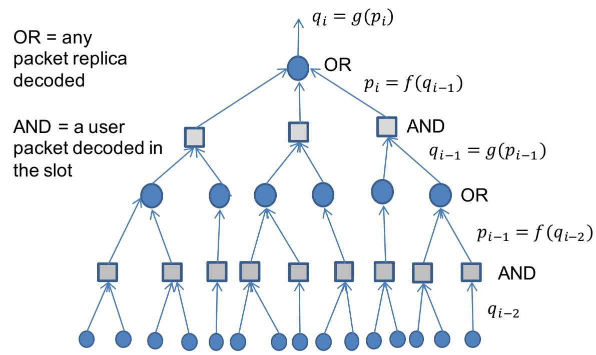

Fig. 5. And-or tree evaluation.

Before giving providing the expressions for p_i and q_i, we introduce the following terminology. Denote the number of edges incident to slot/user node as slot/user node degree. Further, by edge-oriented slot degree distribution ω_j, $j \geq 1$ and $\sum_j \omega_j = 1$, denote the probability distribution that a randomly chosen edge in the graph is connected to a slot node of degree j [9]. Similarly, by edge-oriented user degree distribution λ_k, $k \geq 1$ and $\sum_k \lambda_k = 1$, denote the probability distribution that a randomly chosen edge in the graph is connected to a user node of degree j [9]. Note that λ_k are subject to design of the random access algorithm, and that they implicitly determine ω_j. It could be shown that the probability update in slot node is:

$$p_i = \sum_j \omega_j \sum_{t=0}^{j-1} \pi_{t,j} \binom{j-1}{t} q_{i-1}^t (1-q_{i-1})^{j-t-1},\ i \geq 1, \quad (2)$$

where j is the slot degree, t is the number of interfering users that decreases through iterations via use of IC, $\pi_{t,j}$ is the probability of decoding a user packet in the slot of degree j when t interfering packets have been cancelled, and where the combinatorial term $\binom{j-1}{t}$ stems from the assumption that all colliding user packets in the slot are statistically a-priori

Frameless ALOHA	"standard" case, no capture	capture threshold=1, SNR=10dB	MUD
T_{max}	0.88	2.37 (>1 !)	?

Fig. 6. Theoretically achievable throughput of CCRA.

the same, in terms of probability of being decoded. Here is important to note that the direct influence of the physical layer, i.e., receiver operation, as described in II, is embedded in $\pi_{t,j}$. The probability update in user node is:

$$q_i = \sum_k \lambda_k p_i^{k-1}, \; i \geq 1,$$

with the initial value $q_0 = 1$. Finally, the output of the evaluation is the probability that a user packet is decoded:

$$P_D = 1 - \lim_{i \to \infty} q_i.$$

In Fig. 6 the theoretically achievable throughput figures are summarized for the frameless ALOHA [10], a simple variant of CSA. In frameless ALOHA, the users access slots with a predefined probability, equal for all users and for all slots; each time when a user access the slot, it sends replica of the same data packet. This approach is asymptotically suboptimal, with the throughput limit of 0.88 for the optimal slot access probability. Note that this is below the asymptotically optimal solutions that can reach throughput 1 [3], when all collisions become "completely" decodable via IC and no slot is wasted.

When the capture effect in fading scenarios is taken into account, throughputs well over 1 can be reached, i.e. on average, more than one user can be decoded from a single slot. This is due to the "direct" decodability of collision slots (not just via IC), as noted in Section II-B. The "exploitability" of collision slots can be further boosted with multiuser detection [4] (not applied in this paper).

IV. Simulations

In our (very limited) setting, a number of 3 active users are detected out of a maximum of 50 users. A contending period of 4 slots are used by all active users, such that each sends 2 replicas of their packet. A packet has a size of 100 BPSK symbols, which is exactly the same as a slot size. We consider averaged symbol error rates (SER) in 20 MHz LTE-A random access channel with FFT size $n = 24576$ of which $m = 839$ dimensions are used for CS. Hence, the control overhead for the common random access channel is below 13%. The pilot signalling is equal to [6]. We assume that the delay spread is below $s = 300$ dimensions of which only a set of $k = 3$ paths are actually relevant. Fig. 7 shows the SER performance with SNR=10dB. The graph clearly shows that the performance is only limited by the noise and, hence, collision slots are fully recovered in the iterative process so that the throughput is approximately 0.75.

Eqn. (2) can be derived using the approach similar to [10], [4].

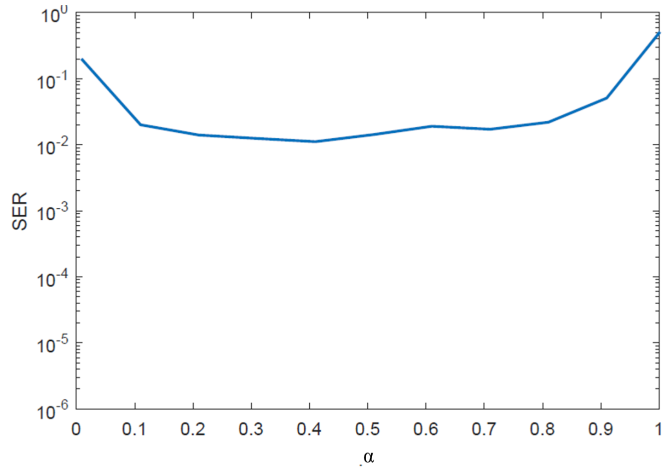

Fig. 7. Average SER of BPSK modulated data for at most 3 (unknown) active one-shot random access users out of a total set of 50 users.

V. Conclusions

In this paper, we analyzed coded random access together with a recently proposed one-shot transmission concept based on a common control channel for 5G massive MTC support. The common control channel is used for activity detection and channel estimation. We presented the overall concept and verified in some initial simulations the promising result that channel estimation is strong enough to resolve the collisions in the iterative process. This shall be further investigated in more advanced 5G scenarios further on.

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